Analytical Model Prediction: Micro-Cutting Tool Forces with the Effect of Friction on Machining Titanium Alloy (Ti-6Al-4V)

Mohd Shahrom Ismail, B.T. Hang Tuah Baharudin, K.K.B. Hon

Abstract—In this paper, a methodology of a model based on predicting the tool forces oblique machining are introduced by adopting the orthogonal technique. The applied analytical calculation is mostly based on Devries model and some parts of the methodology are employed from Amarego-Brown model. Model validation is performed by comparing experimental data with the prediction results on machining titanium alloy (Ti-6Al-4V) based on micro-cutting tool perspective. Good agreements with the experiments are observed. A detailed friction form that affected the tool forces also been examined with reasonable results obtained.

Keywords— dynamics machining, micro cutting tool, Tool forces

I. INTRODUCTION

In recent years, the development of high speed machining (HSM) process becomes essential for rapid processes and production in manufacturing industry. This is necessary due to the requirement of highly accurate and miniaturized size with complex shape applicable to a variation of industries such as aviation, aerospace, biomedicine, medical instruments and communication systems [1]. In the current issue of miniaturization, micro-machining processes are able to produce parts with features varies from several mm to several μm [2]. The demand to manufacture the 3D micro/meso engineering component with a sub-micron surface finish keeps increasing rapidly in industries. Machining of metals is still not completely understood because of the highly non-linear nature of the process and the complex coupling between deformation and temperature fields. Metal cutting can be associated with high temperatures in the tool-chip interface zone and hence, the thermal aspects of the cutting process strongly affect the accuracy of the machining process. Spindle speed played an important role on the cutting tool performance. The deformation process is highly concentrated in a very small zone and the temperatures generated in the deformation zones affect both the tool and the workpiece. The high spindle rotation speed that in the range of 30000 to 100000 revolution per minutes (RPM) give an advantages for the cutting process by increase the metal removing rate, reduced the cutting forces, increase the heat dissipative of thermal energy and high quality surface finished. One of the most important differences on cutting mechanics between high speed machining and conventional machining is that in high speed machining, a formation of chip is most often generated which affects nearly every aspect of high speed machining process, such as cutting force [3], cutting temperature [4] cutting tool wear and life and machined surface quality.

The formation of chip frequently formed in continuous manner for a conventional machining. However it has been observed that serrated chips has formed with adiabatic shear band during HSM of the hardened steel. The earliest effort on predicting the milling machine cutting forces can be regard from the work of [5]. In years later there some authors published their work on the same topic [6-7]. Kline et al. [8] proposed a mechanics of cutting model by taking into account of tool helix angle. Yucesan and Al tintas [9] proposed a cutting forces model by considering such parameters (i.e. frictional coefficient, normal pressure coefficient and chip flow angle). Generally the cutting process variables (i.e. friction, cutting tool geometry, behavior of material etc) have a great effect on cutting tool forces [10]. The characteristics of friction model on cutting operation specifically on the contact between the tool rake and the chip faces are difficult to estimate. Furthermore these factors dominantly influenced the cutting operation output response. Some of researchers determined the friction value by directly measure the normal and friction stress during cutting operation.

II. ANALYTICAL MODEL

A. Estimation of cutting forces model

In this study on the cutting forces was analyzed based on Devries’s model [11], where the oblique cutting technique was employed. Furthermore based on the knowledge of cutting operation such as tool geometry, cutting condition (i.e. depth of cut, feed-rate, and friction factors) leads to the estimation of the cutting force, thrust force and radial force that can be calculated as shown in equation (1).

\[
F_p = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} \text{Cutting Force} \\ \text{Thrust Force} \\ \text{Radial Force} \end{bmatrix}
\]

However equation (1) can be expand on taking into account the cutting process variables as shows in equation (2).

\[
F_p = \begin{bmatrix} c \cos(\phi_n + \beta_n - y_n) \\ c \cos(\beta_n - y_n) \\ c \cos(\beta_n - y_n) \end{bmatrix} \begin{bmatrix} \cos(\eta_n) \cos(\lambda) + c \sin(\eta_n) \sin(\lambda) \\ \cos(\eta_n) + \beta_n - y_n \sin(\lambda) \\ \cos(\eta_n) + \beta_n - y_n \end{bmatrix}
\]

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Following the equation (2) the variables in second term can be referred as shear stress, \( \tau_s \) and width of cut, \( b \). By assuming the friction coefficient, \( \mu \) to be 0.3 the friction angle, \( \beta \) can be expressed throughout equation (3).

\[
\tan \beta = \mu \tag{3}
\]

The shear plane angle, \( \phi_n \) can be derived by employed Merchant model. Apart from that Merchant model act to minimised the magnitude of forces as shows in equation (4).

\[
\phi_n = 45^\circ - \left( \frac{\beta - \gamma_0}{2} \right) \tag{4}
\]

By equated the parameters from cutting and contact conditions and their relationship through equations (3) and (4) the friction angle, \( \beta \) and shear plane angle, \( \phi_n \) are found to be 16.7\(^\circ\) and 23.2\(^\circ\) respectively. Nevertheless the chip flow angle, \( \gamma_c \) can be evaluated by using the Amareggo–Brown model [12]. Generally the chip flow angle, \( \gamma_c \) can predict the angle of the chip flow. The Amareggo–Brown model can be expressed as equation (5).

\[
\eta_c = \tan^{-1} \left( \frac{\tan(\lambda) (\cos(\gamma_n) + \sin(\gamma_n))}{\tan(\phi_n) + \frac{\beta_n}{\gamma_n}} \right) \tag{5}
\]

The shear flow direction, \( \eta_s \) can be workout by using equation (6).

\[
\eta_s = \tan^{-1} \left( \frac{\tan(\lambda) (\cos(\phi_n - \gamma_n) - \tan(\eta_c) \cos(\phi_n))}{\cos(\lambda_n)} \right) \tag{6}
\]

Hence by using the relationship between the equation (5) and (6) the chip flow angle, \( \eta_c \) and shear flow direction, \( \eta_s \) were calculated to be 23.2\(^\circ\) and 6.78\(^\circ\). The feed per tooth of chip flow, \( S_x \) can be calculate by employed equation (7).

\[
S_x = \frac{V_f}{nN} \tag{7}
\]

Where the variables in equation (7) can be derived from the cutting condition during machining operation such as feed rate, \( V_f \), number of flutes, \( n \), and spindle speed, \( N \). Hence the feed per tooth of chip load, \( S_x \) is found to be 4.643 \( \mu \)m/tooth. the maximum uncut chip thickness, \( h_{\text{max}} \) can be calculate as follow as equation (8).

\[
h_{\text{max}} = S_x \sin \left( \cos^{-1} \left(1 - \frac{2a}{D}\right) \right) \tag{8}
\]

Yet equation (8) can be simplified as expressed in equation (9).

\[
h_{\text{max}} = 2S_x \left( \frac{a}{D} \right)^{\frac{1}{2}} \tag{9}
\]

### B. Friction model

The mean coefficient of friction between tool and chip in orthogonal cutting is usually calculated from the measured cutting forces as given with equation (10). The analytical model was verified by comparing the calculated results with experimental data of cutting forces. The data taken from the experiment are based on root mean square (RMS) value to be utilised for a comparison.

\[
\mu = \frac{F_t + F_c \tan(y)}{F_c - F_t \tan(y)} \tag{10}
\]

It is believed that frictional factor played an important role on estimating the cutting tool forces. At the first attempted in this study, the guessed value of friction coefficient is assumed to be 0.3 as been reported by Baharudin [13]. Therefore a further investigation was carried out by varying the friction factor, \( \mu \) via analytical model to minimize the errors on results comparison. The frictional factor is calculated to be 0.39 based on equation (10) by taking into account of a ratio between the cutting forces of experimental results. Meanwhile the guessed frictional factor is varied by increased 20\% of its value in order to study the sensitivity of the tool forces via analytical model. Generally it can be remarked in this paper that 3 types of frictional models were used as part of model validation and frictional effect studies showed in Table I.

#### C. Experimental verification

In this study the analytical model are compared with the micro machining experiment that conducted by [13]. The CNC machining program was generated by adopting the Machining Strategist CAD/CAM software. The 3-axis Kistler dynamometer was used as a standard data acquisition system to extract the tool cutting forces. A slot milling approach was used to generate the cutting tool path based on up-cut slot machining technique with increasing of spindle speed in the ranged of 14000 to 42000 RPM by using constant 1 mm of tool diameter. All machining operations were carried out on a Mikron 700 CNC high speed milling machine.

#### D. Material model

Workpiece material that used in this work is titanium alloy (Ti-6Al-4V) type of material has been used to investigate the effect of cutting parameters on estimating the cutting forces. The workpiece material was set to be a rectangular block with a dimension of (30 x 30 x 30 mm). The titanium alloy (Ti-6Al-4V) is the type of material that has been utilized for several tests in research area due to its unique characteristic. However...
titanium alloy (Ti-6Al-4V) is considered as a difficult to machine materials due to the poor heat conductor where it leads to the increasing of temperatures between the chip-cutting tool contact faces.

### TABLE II
MACHINING PARAMETERS AND MATERIAL PROPERTIES OF TITANIUM ALLOY (Ti-6Al-4V)

<table>
<thead>
<tr>
<th>Machining parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutting speed (mm/min)</td>
<td>130</td>
</tr>
<tr>
<td>Depth of cut, (mm)</td>
<td>0.05</td>
</tr>
<tr>
<td>Tool rake angle, γ (°)</td>
<td>0</td>
</tr>
<tr>
<td>Tool flank angle, α (°)</td>
<td>0</td>
</tr>
<tr>
<td>Tool helix angle, λ(°)</td>
<td>30</td>
</tr>
<tr>
<td>Material properties</td>
<td></td>
</tr>
<tr>
<td>Shear stress, (MPa)</td>
<td>600</td>
</tr>
<tr>
<td>Yield strength (MPa)</td>
<td>880</td>
</tr>
<tr>
<td>Poisson ratio, ν</td>
<td>0.3</td>
</tr>
<tr>
<td>Elastic modulus (GPa)</td>
<td>114</td>
</tr>
<tr>
<td>Elongation at break, (%)</td>
<td>14</td>
</tr>
<tr>
<td>Thermal conductivity (W/m-k)</td>
<td>24.6</td>
</tr>
<tr>
<td>Specific heat capacity (j/kg k)</td>
<td>365</td>
</tr>
</tbody>
</table>

Titanium alloy (Ti-6Al-4V) contain a significant amount of chemical that similar with cutting tool material/coating (i.e. tungsten carbide) and this will accelerates tool wear and tool failure. The material properties of titanium alloy (Ti-6Al-4V) and machining condition are summarized in Table II.

### III. RESULTS

#### A. Model validation

The comparison between experiment and analytical model force for 1 mm micro tool diameter are shown in Table III, where the friction model, $\mu = 0.39$ are taken into account. The relationship between cutting and thrust forces against spindle speed were illustrated on Fig. 1. It can be seen that the results give a reasonable good agreement between the analytical and measured forces obtained. The data reported in Table III indicated the absolute average percentage errors of predicted cutting and thrust forces relative to experimented data approximately around 40.9% and 34%.

It has been observed that the analytical model under estimated the cutting force, $F_c$, to the experiment by 20% of correction factor at 14000 RPM spindle speed. Nevertheless it has been observed that the analytical model estimated the maximum percentage error of cutting and thrust forces relative to the experiment approximately to be 55.03% and 74% at 34000 and 42000 RPM respectively. It been identified that the analytical model over estimated both of cutting and thrust forces compared to the experimental results.

### TABLE III
COMPARISON BETWEEN THE EXPERIMENTAL AND PREDICTION TOOL FORCES WITH FRICTION COEFFICIENT OF 0.39

<table>
<thead>
<tr>
<th>N (RPM)</th>
<th>$F_x^*$ (N)</th>
<th>$F_x$ (N)</th>
<th>%Error</th>
<th>$F_y^*$ (N)</th>
<th>$F_y$ (N)</th>
<th>%Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>14000</td>
<td>3.16</td>
<td>3.77</td>
<td>-19.25</td>
<td>1.11</td>
<td>1.56</td>
<td>-40.99</td>
</tr>
<tr>
<td>18000</td>
<td>3.58</td>
<td>2.93</td>
<td>18.19</td>
<td>1.44</td>
<td>1.22</td>
<td>-6.29</td>
</tr>
<tr>
<td>22000</td>
<td>4.24</td>
<td>2.40</td>
<td>43.45</td>
<td>1.11</td>
<td>1.00</td>
<td>10.41</td>
</tr>
<tr>
<td>26000</td>
<td>3.20</td>
<td>2.03</td>
<td>36.59</td>
<td>1.05</td>
<td>0.84</td>
<td>19.95</td>
</tr>
<tr>
<td>30000</td>
<td>3.64</td>
<td>1.76</td>
<td>51.66</td>
<td>0.98</td>
<td>0.73</td>
<td>25.27</td>
</tr>
<tr>
<td>34000</td>
<td>3.45</td>
<td>1.55</td>
<td>55.03</td>
<td>1.32</td>
<td>0.64</td>
<td>51.19</td>
</tr>
<tr>
<td>38000</td>
<td>3.04</td>
<td>1.39</td>
<td>54.35</td>
<td>1.03</td>
<td>0.58</td>
<td>44.10</td>
</tr>
<tr>
<td>42000</td>
<td>2.45</td>
<td>1.26</td>
<td>48.66</td>
<td>2.01</td>
<td>0.52</td>
<td>74.00</td>
</tr>
</tbody>
</table>

*Experimental results

### TABLE IV
RESULTS VARIATIONS OF FRICTION MODELS ON ESTIMATING THE TOOL FORCES

<table>
<thead>
<tr>
<th>frictional factor, $\mu$</th>
<th>0.3</th>
<th>0.36</th>
<th>0.39</th>
</tr>
</thead>
<tbody>
<tr>
<td>N (RPM)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14000</td>
<td>3.48</td>
<td>1.15</td>
<td>3.70</td>
</tr>
<tr>
<td>18000</td>
<td>2.71</td>
<td>0.89</td>
<td>2.87</td>
</tr>
<tr>
<td>22000</td>
<td>2.21</td>
<td>0.73</td>
<td>2.35</td>
</tr>
<tr>
<td>26000</td>
<td>1.87</td>
<td>0.62</td>
<td>1.99</td>
</tr>
<tr>
<td>30000</td>
<td>1.62</td>
<td>0.54</td>
<td>1.72</td>
</tr>
<tr>
<td>34000</td>
<td>1.43</td>
<td>0.47</td>
<td>1.52</td>
</tr>
<tr>
<td>38000</td>
<td>1.28</td>
<td>0.42</td>
<td>1.36</td>
</tr>
<tr>
<td>42000</td>
<td>1.16</td>
<td>0.38</td>
<td>1.23</td>
</tr>
</tbody>
</table>

### B. Frictional effect on tool force

A sensitivity study has been made by comparing the cutting forces throughout the variation of friction models. The comparisons were made in the analytical model by increased a friction inputs by 20% each. The variation of friction models has been tested as shown in Table IV. From the results it demonstrated that the cutting and thrust forces are increase as the friction increase. In addition, it been noted that the thrust forces for both analytical and experimental are indentified at 14000 RPM for friction factor, $\mu = 0.3$. The characteristic of tool forces are illustrated as shown in Fig. 2. It has been showed that the cutting forces, $F_c$ are increased by average of 6.4% between the transition of coefficient of friction, $\mu$ from 0.3-0.39 and the thrust forces, $F_t$ increased in average of 14% respectively. Nevertheless based on the transition of frictional model, it showed that the tool forces are increased drastically between (0.3-0.36) and stagnant in condition between (0.36-0.39). The forces characteristics showed that both forces are decreasing as the increasing of spindle speed.
The trend of cutting force characteristic showed a good agreement between the analytical model and experiment data. However these responses between predicted and measured cutting forces, $F_x$ started to divergence accordingly toward the incremental of spindle speed. Yet this can be explained that the analytical approach has a limitation to capture the real cutting condition compared to the conducted experiment. Nevertheless the cutting force shows a dominant behavior for both models as it predicted much higher forces compared to thrust forces. The result indicated that analytical models of the thrust force tend to predicted lower forces as the spindle speed increased. However referred to the experiment response the thrust force acted contradicts by estimated a higher forces at higher spindle speed. Throughout this condition it has been assumed that at some point the intermittent cutting condition already occurred during the engagement between the tool and workpiece faces.

From the results, it showed that the cutting force, $F_x$ response from experiment is in steady state condition compared to the analytical model, where it illustrated on much significant reduction of forces as the spindle speed increase accordingly. It been suspected that in the real time experiment, the cutting tools are already worn out and as a result it generated an unstable cutting forces that induced by elevated amount of friction factor. Generally the results showed that the cutting forces are increased proportionately to the increment of frictional values.

In this study the calculated friction model based on equation (10) gives reliable results by estimated a lower average of errors for cutting and thrust forces compared to the other friction models through the validation process.

IV. CONCLUSION

Based on the analysis it can be remarked that the prediction of tool forces showed a good agreement with the experimented data. Even though there are some errors on the comparison studies, it can be regard that the estimation results are sufficient enough for analytical models. Some of the issues can be highlighted in this study as;

1. The analytical model that has been developed throughout this study contributed to the understanding of the tool forces characteristics and trends.
2. The application of the analytical model are for conventional diameter of tool cutter, however throughout this study it proved that it can be applied to the scale of micro-tool size diameter with acceptation results output.
3. Through this study it showed the vital differences between the macro and micro tool machining process that been affected by friction coefficient.
4. Further investigation on the frictional effect can be carried out through finite element model by taking into account of Zorev’s sticking and sliding model.

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REFERENCES
