Abstract—In this paper, the computation of the electrical field distribution around AC high-voltage lines is demonstrated. The advantages and disadvantages of two different methods are described to evaluate the electrical field quantity. The first method is a semi-numerical method using the laws of electrostatic techniques to simulate the two-dimensional electric field under the high-voltage overhead line. The second method which will be discussed is the finite element method (FEM) using specific boundary conditions to compute the two-dimensional electric field distributions in an efficient way.

Keywords—Electrical field, unloaded transmission lines, finite element method, electrostatic images technique

I. INTRODUCTION

In today’s world, an increasing level of sensibility to health and ecological problems is seen. High-voltage overhead lines generate electric and magnetic fields in their neighborhood. The source of the magnetic fields is the currents of phase conductors. The electric field is caused by the high potential of the conductors.

In the course of planning high-voltage AC lines, the electric field quantity around the power lines has to be examined to avoid EMC problems. In installations, the electric field should be limited by the safety distances with respect to the conductors. In different countries, different limiting values exist. International Radiation Protection Association (IRPA) in close collaboration with the World Health Organization (WHO) has derived standards for the time the person stays within the field [1] as below:

1) For workers in the field, the time is given by:
\[ t = \frac{E_{\text{max}}}{9} (10 < E < 30; E \text{ in kV/m}; t \text{ in hours}) \]

2) For the general public, the limiting value is 10 kV/m if the exposure is only a few hours per day and may be increased if the time is shorter. The limiting value in open spaces is 5 kV/m where it is feasible to remain for a longer period, as e.g. recreation zones.

The paper describes how the finite element method [2] can be used to calculate the electric field surrounding a high voltage overhead line.

II. ANALYTICAL APPROACH

Here, an overhead transmission line, situated on a flat area is described. Figure 1 shows a pylon of the overhead line and the configuration of the conductors. A three-phase system was installed only on one side of the pylon. The phases are conductors \( L_1, L_2 \) and \( L_3 \). On the top of the pylon, a grounded conductor \( N \) is installed to protect the lines against lightning strikes. The conductors are considered to be of the same cross section.

Each of the overhead transmission unloaded lines near the earth can be considered as a line charge near a plane conductor. Using the image technique [3] the voltage \( \phi(x,y) \) and the electrical field \( E(x,y) \) can be calculated. In Fig 2, an unload line charge (per unit length) \( \lambda \) has the image line charge (per unit length) \( -\lambda \) and the conducting plane (earth) coincides with the \( xz \)-plane. The voltage \( \phi(x,y) \) due to the two infinitely long parallel line charges \( \lambda \) and \( -\lambda \), as shown in Fig 2, can be written as [3],

\[ \phi(x,y) = -2k\lambda \ln \frac{r_1(x,y)}{r_2(x,y)} \]  \hspace{1cm} (1)

Where,

\[ k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{Nm}^2/\text{c}^2 \]

\[ r_1(x,y) = \sqrt{x^2 + (y-h)^2} \]

\[ r_2(x,y) = \sqrt{x^2 + (y+h)^2} \]

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In the electrostatic case, the relation between \(\mathbf{E}\) and \(\varphi\) has the simple form as below [4]:

\[
\mathbf{E}(x, y) = -\nabla \varphi(x, y)
\]  \hspace{1cm} (2)

In the Cartesian coordinate system, the x and y components of electrical field due to the unloaded single phase of overhead transmission line (Fig.2) can be written as,

\[E_x = -\frac{\partial \varphi(x, y)}{\partial x} = 2k\lambda \left[\frac{x}{x^2 + (y - h)^2} - \frac{x}{x^2 + (y + h)^2}\right]\] \hspace{1cm} (3)

\[E_y = -\frac{\partial \varphi(x, y)}{\partial y} = 2k\lambda \left[\frac{y - h}{x^2 + (y - h)^2} - \frac{y + h}{x^2 + (y + h)^2}\right]\] \hspace{1cm} (4)

Finally, according to the equations (3) and (4), we can calculate the electrical field strength as below,

\[E = \sqrt{E_x^2 + E_y^2} = \frac{\varepsilon_0 k\lambda \lambda^2}{(h^2 - 2hy + x^2 + y^2)(h^2 + 2hy + x^2 + y^2)}\] \hspace{1cm} (5)

III. FINITE ELEMENT METHOD

A two-dimensional finite element model is built lateral to the transmission line. The potential distribution over each element is approximated by a polynomial. Instead of solving the field equations directly, the principle of minimum potential energy is used to obtain the potential distribution over the whole model.

The ratio of the largest size to the smallest size in a finite element model of a transmission line is about 10,000. The circular boundary of the model has a radius of about 100 m, while the radius of the conductors is a few centimeters. Therefore, special attention must be paid to obtain a regular mesh with well-formed elements. This ensures an accurate solution of the field problem. Figure 3 shows part of the finite element model around one of the phase conductors. The change in the size of the elements in the direction away from the conductor can be seen in Fig. 3.

The overhead transmission lines are essentially unbounded. Applying Dirichlet constraints at an outer circular boundary surface at a finite distance from the transmission lines, the potential distribution will not correspond to the real distribution and will be somewhat compressed. The earth is supposed to have an infinite conductivity, yielding the electric field lines to be perpendicular to surface. Therefore, it is not necessary to discretise the ground as indicated in Fig.4.

It has to be mentioned that the unbounded system may be avoided using a special technique dealing with the open boundary problems. This technique is based on the well-known Kelvin-transformation that has recently been accepted to be used in numerical field calculations [5, 6].

IV. ANALYTIC AND FINITE ELEMENT RESULTS AND COMPARISON

The present research has been carried out for 132kV overhead transmission lines between Behbahan substation and Dehdasht substation in Iran. In this network, each phase consists of an ACSR aluminum conductor with radius of 5.8mm and the length 38km.

A. Finite element results

Using the superposition law, the electric field produced by a three phase system will be sinusoidal. In order to calculate the RMS-value of electric field, it is sufficient to know the instantaneous value at two moments, e.g., at \(t=0\) and at \(t=T/4\), where \(T\) is the period of the sinusoidal varying electric field. Assuming a symmetric balanced three phase voltage system, the finite element model is first solved for conductor \(L_1\) at maximum voltage and a second problem is solved for conductor \(L_2\) at zero voltage. This can be done for the component of the field in each arbitrary direction.

The accuracy of the finite element solution depends not only on the element distribution, but also on the order of the finite element approach. Second order solutions of the finite element are compared with the analytical solution for both x-component (horizontal) and y-component (vertical) of the electric field at 1m above ground level in Figure 5 and 6. This comparison is also done in Figure 7 for the electric field strength. In these figures, it can be seen that a second order solution of the finite element method coincides practically with the analytical solution.
B. Analytical results

The electric field and its components were derived for an arbitrary high-voltage overhead transmission line in equations 3, 4, and 5. For this network, we initially obtain $\lambda$ according to the following network characteristics table:

Substituting $\lambda = 2.5335 \times 10^{-6} \left( \frac{\varepsilon}{m} \right)$ and $h=10m$, the electric field and its components due to the high-voltage overhead transmission line of this network, 1m above the ground is:

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>NETWORK CHARACTERISTICS</th>
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<tbody>
<tr>
<td>AC resistivity of conductors at 40°C</td>
<td>$Y = 1.257 \Omega km$</td>
</tr>
<tr>
<td>Network angular frequency</td>
<td>$\omega = 2 \pi f = 6.28 \times 50 \text{ Rad/s}$</td>
</tr>
<tr>
<td>Charge per unit length of conductors</td>
<td>$\lambda = \frac{1}{\omega Y} = \frac{1}{1.257 \times 10^3 \times 314} = 2.5335 \times 10^{-6} \left( \frac{\varepsilon}{m} \right)$</td>
</tr>
</tbody>
</table>

$$E_x = 45.603 \left( \frac{x}{x^2 + 81} - \frac{x}{x^2 + 121} \right) \frac{(\text{KV})}{(\text{m})}$$  \hspace{1cm} (6)

$$E_y = -45.603 \left( \frac{9}{x^2 + 81} + \frac{11}{x^2 + 121} \right) \frac{(\text{KV})}{(\text{m})}$$  \hspace{1cm} (7)

$$E = \frac{1.82412 \times 10^4}{(x^2 + 81)(x^2 + 121)} \frac{(\text{KV})}{(\text{m})}$$  \hspace{1cm} (8)

Two-dimensional plots of equations 6, 7, and 8 are shown in Figures 5, 6, and 7. It is necessary to emphasize that the plots shown in Figures 5 and 6 represent the magnitude of electric field components.

V. CONCLUSION

The topic described above shows that under the given restrictions, the electric field around high voltage overhead lines can be calculated accurately using the finite element method. This procedure can be a useful tool for designers to find the best configuration when several three-phase systems are combined.

A good agreement exists between analytical and finite element methods for calculating the electric field around 132 kV overhead lines.

ACKNOWLEDGMENT

This work supported by Islamic Azad University of Gachsaran.

REFERENCES

