Dempster-Shafer Evidence Theory for Image Segmentation: Application in Cells Images

S. Ben Chaabane, M. Sayadi, F. Fnaiech and E. Brassart

Abstract—In this paper we propose a new knowledge model using the Dempster-Shafer’s evidence theory for image segmentation and fusion. The proposed method is composed essentially of two steps. First, mass distributions in Dempster-Shafer theory are obtained from the membership degrees of each pixel covering the three image components (R, G and B). Each membership’s degree is determined by applying Fuzzy C-Means (FCM) clustering to the gray levels of the three images. Second, the fusion process consists in defining three discernment frames which are associated with the three images to be fused, and then combining them to form a new frame of discernment. The strategy used to define mass distributions in the combined framework is discussed in detail. The proposed fusion method is illustrated in the context of image segmentation. Experimental investigations and comparative studies with the other previous methods are carried out showing thus the robustness and superiority of the proposed method in terms of image segmentation.

Keywords—Fuzzy C-means, Color image, data fusion, Dempster-Shafer’s evidence theory

I. INTRODUCTION

IMAGE segmentation plays an important role in image analysis and computer vision, which is also regarded as the bottleneck of the development of image processing technology for until now there hasn’t been a technique that can handle all the segmentations of different types of image. Recently, color image segmentation attracts more and more attention. It has long been recognized that the human eye can discern thousands of color shades and intensities but only two-dozen shades of gray. The situation often occurs when the objects cannot be extracted using gray scale information but can be extracted using color information. Compared to gray scale, color provides additional information to intensity. People realize that the color is useful or even necessary for pattern recognition and computer vision. Also the acquisition and processing hardware for color image becomes more and more available for dealing with the problem of computation complexity caused by the high-dimensional color space. Hence, color image processing is becoming increasingly prevalent nowadays. In most of the existing color image segmentation approaches, a region denotes a similar color region. Monochrome image segmentation techniques can be extended to color image, such as histogram thresholding [4] [5] [15], clustering, region growing, edge detection, fuzzy logic [12] [13], and neural networks [9] [14], by using RGB or their transformations (linear/non-linear) as shown in fig.1 [6].

Each color representation has its advantages and disadvantages [20]. There is still no color representation that can dominate the others for all kinds of color images yet. The major problem of linear color spaces the high correlation of the three components, which makes the three components dependent upon each other and strongly associated with intensity. Hence, linear spaces are very difficult to discriminate highlights, shadows and shadings in color images.

In our study, we have chosen to work only with the three primitive colors (R, G and B) given by the sensor. Each color plane is considered as an information source which can be imprecise or uncertain. In this context, data fusion techniques based on exploiting redundant and complementary information from different sources appears an interesting approach for segmenting image. First, fuzzy c-means (FCM) clustering [1] [19], is used to automatically determining of mass functions. Each pixel is then characterized by its membership values in clusters or classes or still hypotheses. Once, the mass functions are determined for each image to be fused, the Dempster-Shafer evidence theory is applied to obtain the final segmentation result.

The rest of the paper is organized as follows. Section 2 presents the Dempster-Shafer evidence theory. In section 3, we describe the proposed method. The experimental results and discussions are in section 4. Finally, conclusions are presented in section 5.

II. SOME FUNDAMENTALS OF DEMPSTER-SHAFER’S EVIDENCE THEORY

The Dempster-Shafer theory (DS), also known as the theory of belief functions, is a generalization of the Bayesian theory of subjective probability.

Whereas the Bayesian theory requires probabilities for each question of interest, belief functions allow us to base belief degrees for one question on probabilities to a related question. These degrees of belief may or may not have the mathematical
properties of probabilities. This theory is a mathematical theory of evidence [2] based on belief functions and plausible reasoning, which is used to combine separate pieces of information (evidence) to calculate the probability of an event.

The theory was developed by Arthur P. Dempster [2] and generalized by Glenn Shafer [17]. She allows one to consider the confidence one has in the probabilities assigned to the various outcomes.

Let \( X \) be the universal set, the set of all states under consideration. The power set \( \mathcal{P}(X) \) is the set of all possible sub-sets of \( X \), including the empty set, \( \phi \). For example, if:

\[
X = \{a, b\}
\]

Then:

\[
P(X) = \{\phi, a, b, X\}
\]

\[
m(\phi) = 0
\]

The masses of the remaining members of the power set add up to a total of 1:

\[
\sum_{A \in \mathcal{P}(X)} m(A) = 1
\]

The mass \( m(A) \) of a given member of the power set, \( A \), expresses the proportion of all relevant and available evidence that supports the claim that the actual state belongs to \( A \) but no to particular subset of \( A \). The value of \( m(A) \) pertains only to the set \( A \) and makes no additional claims about any subsets of \( A \), each of which has, by definition, its own mass.

From the mass assignments, the upper and lower bounds of a probability interval can be defined. This interval contains the precise probability of a set of interest (in the classical sense), and is bounded by two non-additive continuous measures called belief (or support) and plausibility:

\[
\text{bel}(A) \leq P(A) \leq \text{pl}(A)
\]

The belief \( \text{bel}(A) \) for a set \( A \) is defined as the sum of all the masses of (not necessarily proper) subsets of the set of interest:

\[
\text{bel}(A) = \sum_{B \subseteq A} m(B)
\]

The plausibility \( \text{pl}(A) \) is the sum of all the masses of the sets \( B \) that intersect the set of interest \( A \):

\[
\text{pl}(A) = \sum_{B \cap A \neq \phi} m(B)
\]

The two measures are related to each other as follows:

\[
\text{pl}(A) = 1 - \text{bel}(\bar{A})
\]

It follows from the above that it is necessary to know one of the three (mass, belief, or plausibility) to deduce the other two, though it remains to know the values for many sets in order to calculate one of the other values for a particular set.

The problem is how to combine two independent sets of mass assignments. The original combination rule, known as Dempster's rule of combination, is a generalization of Bayes' rule. This rule strongly emphasizes the agreement between multiple sources and ignores all the conflicting evidence through a normalization factor. Use of that rule has come under serious criticism as far as the significant conflict in the information is encountered.

Specifically, the combination (called the joint mass) is calculated from the two sets of masses \( m_1 \) and \( m_2 \) in the following manner:

\[
m_{1,2}(\phi) = 0
\]

\[
m_{1,2}(A) = \frac{1}{1 - K} \sum_{B \cap C \neq \phi} m_1(B) m_2(C)
\]

Where

\[
K = \sum_{B \cap C \neq \phi} m_1(B) m_2(C)
\]

Where \( K \) is a measure of the amount of conflict between the two mass sets. The normalization factor, \( 1 - K \), has the effect of completely ignoring conflict and attributing any mass associated with conflict to the null set. Consequently, this operation yields counterintuitive results in the face of significant conflict in certain contexts.

Often used as a method of sensor fusion, Dempster-Shafer theory is based on two ideas: obtaining degrees of belief for one question from subjective probabilities for a related question, and Dempster's rule [17] for combining such degrees of belief when they are based on independent items of evidence. In essence, the degree of belief in a proposition

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**Color Segmentation Approaches**

<table>
<thead>
<tr>
<th>Monochrome Segmentation Approaches</th>
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<tr>
<td>Histogram Thresholding,</td>
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**Color Spaces**

- RGB,
- YIQ,
- YUV,
- I1I2I3,
- HIS,
- Nrgb,
- CIEL*u*v*,
- CIEL*a*b*,
- Hybrid color space

Fig. 1 commonly used color image segmentation approaches [6]
depends primarily upon the number of answers (to the related questions) containing the proposition, and the subjective probability of each answer. There are also rules of combination that contributed to the general assumptions about the data.

In this formalism a degree of belief (also referred to as a mass) is represented as a belief function rather than a Bayesian probability distribution. Probability values are assigned to sets of possibilities rather than single events: their appeal rests on the fact they naturally encode evidence in favor of propositions.

Shafer's framework allows for belief about propositions to be represented as intervals, bounded by two values, belief (or support) and plausibility: belief ≤ plausibility

Belief in a hypothesis is constituted by the sum of the masses of all sets enclosed by it (i.e. the sum of the masses of all subsets of the hypothesis). It is the amount of belief that directly supports a given hypothesis at least in part, forming a lower bound. Plausibility is 1 minus the sum of the masses of all sets whose intersection with the hypothesis is empty (equivalently, it is the sum of the masses of all sets whose intersection with the hypothesis is not empty). It is an upper bound on the possibility that the hypothesis could possibly happen, i.e. it "could possibly happen" up to that value, because there is only so much evidence that contradicts that hypothesis.

Beliefs corresponding to independent pieces of information are combined using Dempster’s rule of combination (see equation 4, in the case of j informations), which is a generalization of the special case of Bayes’ theorem where events are independent (There is as yet no method of combining non-independent pieces of information). Note that the probability masses from propositions that contradict each other can also be used to obtain a measure of how much conflict there is in a system. This measure has been used as a criterion for clustering multiple pieces of seemingly conflicting evidence around competing hypotheses.

A. Fuzzy clustering

The Fuzzy c-means is an unsupervised clustering algorithm which can be applied successfully to several problems involving feature analysis, clustering and classifier design in fields such as astronomy, chemistry, geology, image analysis, medical diagnosis, shape analysis, target recognition and image segmentation [3][8][11][19].

The FCM algorithm minimizes the objective function for the partition of data set, $X = [x_1, x_2, ..., x_d]^T$, given by:

$$J_m(u,v) = \sum_{i=1}^{c} \sum_{k=1}^{d} u_{ik}^m ||x_k - v_i||^2$$

In this equation, d is the number of samples in the vector X, c is the number of clusters (or classes) (1 ≤ c ≤ d), $u_{ik}$ is the element of the partition matrix $U$ of size $(c \times d)$ containing the membership function, $v_i$ is the center of the $i$th cluster, and m is a weighting factor that controls the fuzziness of the membership function.

The matrix U is constrained to contain elements in the range $[0,1]$ such that $\sum_{i=1}^{c} u_{ik} = 1$, for each $k \in [1,d]$. The norm $||x_k - v_i||$ is the distance between the sample $x_k$ and the centers of classes $v_i$ ($i \in [1,c]$).

In the framework of the segmentation of a multi-textured image of size $(N \times M)$, the vector X contains all the gray level of the image, scanned line by line, i.e. $d=NM$. The fuzzy c-means algorithm performs the partition of the vector X into c fuzzy subsets where $u_{ik}$ represents the membership of $x_k$ in class i.

The FCM clustering technique can be summarized by the following steps [1][18]:

1. Initialization (Iteration 0)
2. Scan the image line by line to construct the vector X containing all the gray level of the image.

III. THE PROPOSED METHOD

Data fusion process consists of combining information from different sources in order to improve the decision process [7]. The fusion can be achieved either in the centralized manner or in a distributed manner. In the context of image segmentation, the decision concerns the classification of a three of pixels (representing the same physical point p) coming from three images (representing the three component of color image) into one class $C_i$ of classes set $\Omega = \{C_i\}_{i=1}^{N}$. Therefore, the main idea of our paper is the utilization of the DS theory for fused one by one the pixels coming from the three images. In this study, the method of generating mass functions is based on the concept of fuzzy logic. For a given pixel, to obtain the membership function of its corresponding gray level, an unsupervised fuzzy clustering algorithm (FCM) was used.

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Randomly initialize the centers of the classes vector $V^{(0)}$.

From the iteration $t=1$ to the end of the algorithm:

Step 2: Calculate the membership matrix $U^{(t)}$ of element $i_k$ ($i \in [1, c]$, $k \in [1, d]$) using:

$$u_{ik} = \left( \sum_{j=1}^{c} \left( \frac{\left\| x_k - v_j \right\|}{\left\| x_k - v_i \right\|} \right)^2 \right)^{-1}$$  

(14)

Step 3: Calculate the vector $V^{(t)} = [v_1, v_2, \ldots, v_c]$ using:

$$v_j = \frac{\sum_{k=1}^{d} u_{ik} x_k}{\sum_{k=1}^{d} u_{ik}}$$  

(15)

Step 4: Convergence test:

If $\left\| V^{(t)} - V^{(t-1)} \right\| > \varepsilon$, then increment the iteration $t$, and return to the Step 2, otherwise, stop the algorithm. $\varepsilon$ is a chosen positive threshold.

B. Mass function determination using FCM

Let $\Omega$ represents the finite set of regions $R_u$ where $R_u$ replaces the previous (or $H_i$), $\Omega = \{R_u\}$ for $u=1, 2, \ldots, U$.

Each color plane (R, G, and B) is assimilated to an information source $S_l$ for $l = 1, \ldots, L$. Let us consider a basic belief assignment $m^{S_l}$ defined as:

$$m^{S_l} : 2^\Omega \rightarrow [0,1]$$  

(16)

With

$$m^{S_l}(\phi) = 0$$  

(17)

$$\sum_{A \in \Omega} m^{S_l}(A) = 1$$  

(18)

In the use of evidence theory for image segmentation, the determination of mass functions is delicate but a key point. In the present study, masses of simple hypotheses $H_i$ are directly obtained from the membership functions $\mu_i(x_k)$ of the gray level $x_k$ to cluster $i$ as follows:

$$m_i(x_k) = \mu_i(x_k)$$  

(19)

The advantage of Dempster-Shafer’s theory lies in representing uncertainty by means of belief on the whole frame of discernment. This basic belief assignment allows defining $m^{S_l}(\Omega)$ with the following equation:

$$m_i(\Omega) = \prod_{j} (1 - m_j(x_k))$$  

(20)

Therefore, a physical point $p$ is associated to three different grey levels $x_1$, $x_2$ and $x_3$ from images $R$, $G$ and $B$ respectively. For each pixel $x_1$, $x_2$ and $x_3$, it corresponds the mass function $m_{s_1}$, $m_{s_2}$ and $m_{s_3}$ respectively. Once these extended are made, the three distributions $m_{s_1}$, $m_{s_2}$, and $m_{s_3}$ are defined as one the same space of discernment. The combination of Dempster is used to evaluate the final distribution of mass $m_{s_{1,2,3}}$ such as:

$$m_{s_{1,2,3}} = m_{s_1} \oplus m_{s_2} \oplus m_{s_3}$$  

(21)

After calculating the orthogonal sum of the mass functions for the three images, the decisional procedure for classification purpose consists in choosing one of the most likely hypothesis $H_i$.

IV. EXPERIMENTAL RESULTS

To evaluate the proposed approach, tests were first realized on simulated images (fig. 2). The images contain two regions. In fact, some obtained results are given using the above presented method. To get a better insight into the actual ability of the proposed approach, a comparison was made between with conventional algorithms such as those based on the Assumption of Gaussian Distributions (AGD) [16] and those based on the Euclidean Distance (ED) [10] for determining the mass functions. Table 1 and figure 2 shows an example of comparison between these three approaches based on the segmentation sensitivity $Sens\%$ (see equation 22).
With: Sens, Npcc, NxM correspond respectively to the segmentation sensitivity (%), Number of correctly classified pixels and dimension of the image.

In the following example and in the goal of the automatically detection of cells (see figure 4), the decision has been made using the criterion of maximum mass function. The proposed segmentation algorithm has been applied to a cell image in order to illustrate the methodology. The original image represented in RGB space is shown in fig. 4(a). The image contains two regions (C=2).

The results of R, G and B components by the FCM are shown in fig 4(b), 4(c) and 4(d) respectively. Fig 4(e) is the final result after the first stage (FCM) and the second stage (DS). In the first stage, the result segmentation is got with a fuzzy factor $m = 2$ and $\varepsilon = 10^{-5}$. We have noted that 16.94 %, 17.58% and 13.34 % of pixels have been unclassified for the three primitive colors R, G and B, respectively. These unclassified pixels reflected the influence of lack information and high correlated of the three primitive colors (R, G and B) of final segmentation.

### TABLE I

<table>
<thead>
<tr>
<th>Sens%</th>
<th>Euclidian Distance (ED)</th>
<th>Assumption of Gaussian Distributions (AGD)</th>
<th>Fuzzy C-Means (FCM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image 1</td>
<td>68.23</td>
<td>79.66</td>
<td>88.92</td>
</tr>
<tr>
<td>Image 2</td>
<td>66.84</td>
<td>71.15</td>
<td>89.45</td>
</tr>
<tr>
<td>Image 3</td>
<td>72.56</td>
<td>83.19</td>
<td>87.25</td>
</tr>
<tr>
<td>Image 4</td>
<td>85.11</td>
<td>88.91</td>
<td>96.68</td>
</tr>
<tr>
<td>Image 5</td>
<td>75.42</td>
<td>76.86</td>
<td>90.15</td>
</tr>
<tr>
<td>Image 6</td>
<td>59.85</td>
<td>65.43</td>
<td>89.78</td>
</tr>
<tr>
<td>Image 7</td>
<td>66.78</td>
<td>79.33</td>
<td>88.79</td>
</tr>
<tr>
<td>Image 8</td>
<td>75.48</td>
<td>77.85</td>
<td>99.63</td>
</tr>
<tr>
<td>Image 9</td>
<td>83.54</td>
<td>93.88</td>
<td>96.88</td>
</tr>
</tbody>
</table>

Fig. 3 Plot of the segmentation sensitivity for the 9 color images using the ED, DGA and FCM approaches for determining the mass functions.

\[
Sens = \frac{Npcc}{NxM} \times 100, \quad (22)
\]
In figure 4(e), it is observed that the two regions are well brought out, showing that the complementary information provided by the three images was well exploited by the fusion algorithm.

The segmentation errors have been largely reduced while combining the three images through the use of DS fusion approach. Indeed only 0.37% of pixels have been unclassified. The segmented result shows the presence of two classes. Consequently, a comparison of classification error rate between using the three approaches (see figure 3 and table 1), for determining the mass functions in Dempster-Shafer evidence theory demonstrates a significant reduction in classification error rate when using FCM.

In fact, it is observed by using both the FCM algorithm and the evidence theory for the color image segmentation, that the segmented regions are rather homogeneous which makes it possible to do an accurate measurement of cells volumes.

V. CONCLUSION

In this paper, we have presented a color image segmentation approach based on Dempster-Shafer evidence theory. The key point of this approach is automatically determining the mass function by applying Fuzzy c-means clustering. The paradigm for deriving mass distributions associated with the images to be fused has been described in detail. The work presented shows that segmentation based on
evidential data fusion is robust in the sense that only the membership functions of each gray level to each cluster of images are needed. In the future works, the proposed fusion method will be applied to a larger class of images. On the other hand, the available information (mass distribution) will be used to evaluate the quality of the segmentation.

REFERENCES


BIBIOGRAPHIES

Salim Ben Chaabane born in 1979 in La Chebba (Tunisia), he received the BSc degree in Electrical Engineering from the High school of sciences and techniques of Tunis, the DEA degree in Automatic from same school respectively in 2004 and 2006. His research interests are focused on signal Processing, image processing, classification, segmentation and data fusion.

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