Abstract—A variable structure model reference adaptive control (VS-MRAC) strategy for active steering assistance of a two wheel steering car is proposed. An ideal steering system with fixed properties and moving on an ideal road is used as the reference model, and the active steering assistance system is forced to attain the same behavior as the reference model. The proposed system can treat the nonlinear relationships between the side slip angles and lateral forces on tire, and the uncertainties on friction of the road surface, whose compensation are very important under critical situations. Simulation results show improvements on yaw rate and side slip.

Keywords—Variable Structure, Adaptive Control, Model reference, Active steering assistance.

I. INTRODUCTION

MOTION control of autonomous vehicles has many problems to be considered according to the freedom of motion. Fig. 1 shows the dynamics model of a four wheel vehicle which is assumed to be a rigid body. It has six degrees of freedom. F1: up and down motion along to z-axis, F2: lateral motion along to a y-axis, F3: longitudinal motion along to an x-axis, F4: rolling motion around an x-axis, F5: pitching motion around a y-axis and F6: yawing motion around a z-axis. Among them, in order to consider the automatic steering system, it will be derived the mathematical model with three degree of freedom i.e. F2: lateral motion along to a y-axis, F3: longitudinal motion along to an x-axis, and F6: yawing motion around a z-axis, because it is the most important system for motion control of autonomous vehicle. Although the concept of a four-wheel steering (4WS) system has been introduced to enhance vehicle handling [1, 2], the need for 4WS is not so obvious. Some researchers have shown disadvantages on 4WS vehicles [3, 4]. In this paper, we will employ two–wheel steering (2WS) vehicle for cost and implementation issues in active car steering. However, it is generally fair to say that 4WS vehicle is easier to handle than the 2WS vehicle.

In this paper we investigate the Variable Structure Model Reference Adaptive Controller (VS-MRAC) [5], for a 2WS system.

The basic concept of the variable structure control is that of sliding mode control. Switching control functions are generally designed to generate sliding surface, or sliding modes, in the state space [6]. When this is attained the switching functions keep the trajectory on the sliding surfaces and the closed loop system becomes insensitive, to a certain extent, to parameter variations and disturbances.

A number of other control strategies for passenger vehicle are given in references [7, 8]. In addition, LMI approach to 4WS vehicle is investigated in reference [9].

This paper is organized as follows: in section II we introduce the model of steering system, section III subscribe the VS-MRAC controller and using steering system in this controller, section IV we modify our matrices that get in section III and show the simulation results in ICY and WET roads and in section V the conclusion is expressed.

II. DYNAMIC MODEL OF STEERING MOTION

The feature of car steering dynamics in horizontal plane is described by Fig. 2 [10, 11]. In the horizontal plane of Fig. 2 an inertial fixed coordinate system $(X, Y)$ is shown together with a vehicle fixes coordinate system $(x, y)$ that is rotated by a $\psi$. In the dynamic equations the $r = \psi$ will appear as a state variable. Assuming that:

$$\beta_f = \beta_{f1} = \beta_{f2} = \delta - \beta - \frac{I_f}{r/V},$$

$$\beta_r = \beta_{r1} = \beta_{r2} = -\beta + \frac{I_r}{r/V},$$

and $|\beta_f| << 1, |\beta_r| << 1, |\delta| << 1$, then the two wheel model [12]. Fig. 3 can be regarded as the equivalent model to the four wheel model (Fig. 2).
The side forces \( f_y = 2Y_f, f_x = 2Y_r \) are projected through the steering angle into chassis coordinate \((x, y)\), where they appear as forces \( f_y', f_x \), and the torque \( m_z \) around a \( z\)-axis which is pointing upward from the center of gravity \((P)\).

\[
\begin{bmatrix}
  f_x \\
  f_y \\
  m_z
\end{bmatrix} =
\begin{bmatrix}
  -\sin \delta & 0 \\
  \cos \delta & 1 \\
  l_f \cos \delta & -l_r
\end{bmatrix}
\begin{bmatrix}
  f_f' \\
  f_r'
\end{bmatrix}
\tag{1}
\]

Via the dynamics model the forces cause state variables \( \beta, V, r \). The equations of motions for three degrees of freedom in the horizontal plane are:

1. for longitudinal motion
   \[
   -mV(\beta + r) \sin \beta + mV \cos \beta = f_x \tag{2}
   \]
2. for lateral motion
   \[
   mV(\beta + r) \cos \beta + mV \sin \beta = f_y \tag{3}
   \]
3. for yaw motion
   \[
   I_r = m_z \tag{4}
   \]

It is obtained that from (2) to (4)

\[
\begin{bmatrix}
  mV(\beta + r) \\
  mV' \\
  l_r
\end{bmatrix} =
\begin{bmatrix}
  -\sin \beta & \cos \beta & 0 \\
  \cos \beta & \sin \beta & 0 \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  f_x' \\
  f_y' \\
  m_z
\end{bmatrix}
\tag{5}
\]

The side force \( f_y' \) is known to be the nonlinear function of the tire side slip angles \( \beta_f, \beta_r \), i.e.

\[
 f_y = f_y(\beta_f) + f_y(\beta_r) \tag{6}
\]

Two wheel models (5) and (6) are nonlinear and we will introduce the additional assumptions \([11]\) as follows.

(Assumption 1): the sideslip angle \( \beta \) is assumed to be small. Then (5) become

\[
mV(\beta + r) =
\begin{bmatrix}
  -\beta & 1 & 0 \\
  1 & \beta & 0 \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  f_x' \\
  f_y' \\
  m_z
\end{bmatrix}
\tag{7}
\]

(Assumption 2): the velocity is constant, \( V = 0 \). Then, the second row of (7) is eliminated.

\[
mV(\beta + r) =
\begin{bmatrix}
  -\beta & 0 \\
  0 & 1
\end{bmatrix}
\begin{bmatrix}
  f_x' \\
  m_z
\end{bmatrix}
\tag{8}
\]

The velocity \( V \) is treated as an uncertain constant parameter.

(Assumption 3): the nonlinear characteristic of (6) is approximated by nominal value of the tangent at \( \beta_f = \beta_r = 0 \) and small nonlinear functions i.e.

\[
 f_y(\beta_f) = c_f(\mu(\beta_f + \Delta_1(\beta_f, r)) \),
 f_r(\beta_r) = c_r(\mu(\beta_r + \Delta_2(\beta_r, r)),
\tag{9}
\]

Where \( \Delta_i(\beta, r), (i=1,2) \) is \( C^\infty \) nonlinear function and \( \|\Delta_i(\beta, r)\| \ll 1 \) \([13,14]\). Typical experimental values of \( \mu \) \([11,15,16]\) are

\[
\mu = 1 \quad \text{dryroad}
\]
\[
\mu = 0.7 \quad \text{wetroad}
\]
\[
\mu = 0.3 \quad \text{icyroad}
\tag{10}
\]

The steering model follows from (7) to (9) and using (1) as

\[
\begin{bmatrix}
  mV(\beta + r) \\
  l_r
\end{bmatrix} =
\begin{bmatrix}
  -\frac{\rho}{l_r} & 1 & 0 \\
  -\frac{\rho}{l_r} & -\frac{\rho}{l_r} + \Delta_1(\beta, r)
\end{bmatrix}
\begin{bmatrix}
  \frac{l_r}{V} - \Delta_2(\beta, r)
\end{bmatrix}
\tag{11}
\]

The uncertain parameters in this model are \( m, I, V \) and \( \mu \). Solving (11) for \( \beta \) and \( r \) rearranging terms yields the nonlinear state space model:
The control input r to the plant, is generated introducing control law

\[ u = \sigma (\beta X + r) \]  

Where \( n \)-dimensional raw feedback vector \( \beta \), with the elements \( \beta_i \) are adjust using VS approach by designing switching functions \( \hat{\beta}_i \) as described in following.

\[ \beta_i = -\hat{\theta}_i \text{sgn} \left( \beta^T P e \theta \right) \]  

Where

\[ A_m^T P + P A_m = -Q_0 \]  

Is a Lyaponov equation with Lyaponov function \( V = e^T P e \). And \( Q_0 \) is a positive definite symmetric matrix. Also switching function \( \sigma \) is define as follow

\[ \sigma = -q \text{sgn} \left( \beta^T P e u^0 \right) \text{sgn} (q^*) \]  

You can refer to [5] for the proof of equations.

IV. VS-MRAC FOR STEERING SYSTEM - SIMULATION RESULTS

In steering system we have three uncertainties, \( \mu \) as adhesion of road and \( c_f, c_r \), as cornering stiffness of front and rear tires respectively. Although these are our uncertainties, we know the range of variation of them. For adhesion of road we have

\[ 0 \leq \mu \leq 1 \]  

And also for cornering stiffness of front and rear tires we have

\[ 25 \text{KN/rad} \leq c_f \leq 150 \text{KN/rad} \]  

\[ 25 \text{KN/rad} \leq c_r \leq 150 \text{KN/rad} \]  

For showing our results we let these parameters as follow for model and vehicle cases. In model case all uncertainties are imagine. So for model case we have:

\[ m = 1170 \text{Kg}, l_f = 1.4m, l_r = 1.8m \]  

\[ V = 30 \text{m/s} = 108 \text{Km/hr}, i = 1.341 \text{m}^2 \]  

\[ c_f = 6 \text{KN/rad}, c_r = 10 \text{KN/rad}, \mu = 1 \]  

So our model matrix according to equations (12) and (13) is:

\[ A_m = \begin{bmatrix} -0.4558 & -0.9906 \\ 6.2971 & -0.6436 \end{bmatrix}, \quad b_m = \begin{bmatrix} 0.1709 \\ 3.7094 \end{bmatrix}, \quad C_m = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D_m = \begin{bmatrix} 0 \end{bmatrix} \]  

For vehicle case in icy road we have

\[ m = 1170 \text{Kg}, l_f = 1.4m, l_r = 1.8m \]  

\[ V = 30 \text{m/s} = 108 \text{Km/hr}, i = 1.341 \text{m}^2 \]  

\[ c_f = 25 \text{KN/rad}, c_r = 25 \text{KN/rad}, \mu = 0.3 \]  

So we have
And also for vehicle case in wet road we have:

\[
m = 1170 Kg \quad l_f = 1.4 m \quad l_r = 1.8 m
\]

\[
V = 30 \frac{m}{s} = 108 \frac{Km}{hr} \quad i = 1.341 m^2
\]

\[
c_f = 25 KN/\text{rad} \quad c_r = 25 KN/\text{rad} \quad \mu = 0.7
\]

And our matrices are as follow:

\[
A = \begin{bmatrix} -0.4274 & -0.9957 \\ 2.8681 & -0.5427 \end{bmatrix}, b = \begin{bmatrix} 0.2137 \\ 4.6368 \end{bmatrix},
\]

\[
C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D = [0]_{2 \times 1}
\]

And our matrices are as follow:

\[
A = \begin{bmatrix} -0.9972 & -0.99 \\ 6.6933 & -1.2663 \end{bmatrix}, b = \begin{bmatrix} 0.4986 \\ 10.8192 \end{bmatrix},
\]

\[
C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D = [0]_{2 \times 1}
\]

In this example our steering input shows in Fig. 4. Now we consider the results in icy and wet roads. In all figures green curve is model, blue curve is MRAC and red curve is Passive (open loop) control.

**V. CONCLUSION**

According to the Figs. 5 to 12 we see that the MRAC controller follow the model case in good condition. And we see that errors of this following are small and we can ignore them. On the other words, errors between model and MRAC are going to zero. Then we achieve our target and controller provide good situation for driver.

**REFERENCES**


