New Technologies for Modeling of Gas Turbine Cooled Blades

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Abstract - In contrast to existing methods which do not take into account multiconnectivity in a broad sense of this term, we develop mathematical models and highly effective combination (BEM and FDM) numerical methods of calculation of stationary and evanescent stationary temperature field of a profile part of a blade with convective cooling (from the point of view of realization on PC). The theoretical substantiation of these methods is proved by appropriate theorems. For it, converging quadrature processes have been developed and the estimations of errors in the terms of A.Zi灯具 continuity modules have been received. For visualization of profiles are used: the method of the least squares with automatic conjecture, device spline, smooth replenishment and neural nets. Boundary conditions of heat exchange are determined from the solution of the corresponding integral equations and empirical relationships. The reliability of designed methods is proved by calculation and experimental investigations heat and hydraulic characteristics of the gas turbine 1st stage nozzle blade

Keywords—multiconnected systems, method of the boundary integrated equations, splines, neural networks.

1. INTRODUCTION

The development of aviation gas turbine engines (AGTE) at the present stage is reached, in main, by assimilation of high values of gas temperature in front of the turbine ($T_\text{L}$). Despite of the fact, that the activities on temperature increase are conducted in several directions. However, assimilation of high $T_\text{L}$ in AGTE is reached by refinement of cooling systems of turbine hot details, and first of all, nozzle and working blades. It is especially necessary to underline, that with increase the requirement to accuracy of eventual results will increase. In other words, at allowed (permissible) in AGTE metal temperature ($T_\text{lim}$ = (1100...1300 K), $t_\text{lim}$ the absolute error of temperature calculation should be in limits (20-30 K), that is no more than 2-3%. It is difficult to achieve. In skew fields of complicated shape with various cooling channels, quantity and arrangement having a complex configuration, that is in multiply connected areas with variables in time and coordinates by boundary conditions such problem solving requires application of modern and perfect mathematical device.

2. PROBLEM FORMULATION

In classical statement a heat conduction differential equation circumscribing in common case non-stationary process of distribution of heat in many – dimensional area (an equation of Fourier-Kirch noteworthy) has a kind [1]:

$$\frac{\partial}{\partial t} \left( \rho C_v T \right) = \nabla \cdot ( \lambda \nabla T ) + q,$$

where $C_v$ and $\rho$ - accordingly material density, thermal capacity and heat conduction, $q$ - internal source or drain of heat, and $T$ - required temperature.

By results of researches, it has been established [2], that the temperature condition of a profile part of a blade with radial cooling channels can with a sufficient degree of accuracy be determined, as two-dimensional. Besides if to suppose a constancy of physical properties, absence of internal sources (drains) of heat, then the temperature field under fixed conditions will depend only on the shape of a skew field and on distribution of temperature on skew field boundaries. In this case, equation (1) will look like:

$$\Delta T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

(2)

When determining particular temperature fields in gas turbine elements are more often set boundary conditions of the third kind describing heat exchange between a skew field and an environment on the basis of a hypothesis of a Newton-Riemann. In that case, these boundary conditions will be recorded as follows:

$$\alpha_0(T_0 - T) = \lambda \frac{\partial T}{\partial n}$$

(3)

Characterizes quantity of heat transmitted by convection from gas to unit of a surface of a blade and assigned by heat conduction in a skew field of a blade:

$$- \lambda \frac{\partial T}{\partial n} = \alpha_0(T_0 - T)$$

(4)

Characterizes quantity of a heats assigned by a convection of the chiller, which is transmitted by heat conduction of the material of a blade to the surface of cooling channels; where $T_0$ -- temperature of environment at $i=0$; $T_0$ -- temperature of environment at $i \in \mathbb{N}$ (temperature of the chiller), where $\mathbb{N}$- quantity of outlines; $T_0$ -- temperature on an outline $\gamma_i$ at $i=0$, (outside outline of blade); $T_0$ -- temperature on an outline $\gamma_i$ at $i = 1, \mathbb{N}$ (outlining of cooling channels); $q$ - heat transfer factor from gas to a surface of a blade (at $i = 0$) -- heat transfer factor from a blade to the cooling air at $i = 1, \mathbb{N}$; $\lambda$ -- thermal conductivity of the material of a blade; $n$ - external normal on an outline of researched area.

3. PROBLEM SOLUTION

At present for the solution of this boundary value problem (2)-(4) most broad distribution have received four numerical methods – methods of final differences (MFD), finite element method (FEM), probabilistic method (or method Monte-Carlo) and method of boundary integral equations (BIEM) (or its discrete analog - a method of boundary element (BEM)).

Let's consider BIEM application for the solution of problem (2)-(4) [1-3, 14].

3.1. In contrast to [4] we offer to decide the given boundary value problem (2)-(4) as follows [1-3, 14]. We suppose, that distribution of temperature $T(x,y)$ we locate as follows:

$$T(x,y) = \int_{\gamma_i} \frac{\rho \gamma_i}{n R^2} ds,$$

(5)
where \( \Gamma = \bigcup_{i=0}^{n} \gamma_i \) - smooth closed Jordan curve; \( M \) - quantity of cooled channels; \( \rho = \bigcup_{i=0}^{n} \rho_i \) - density of a logarithmic potential uniformly distributed on \( \gamma_i \), \( S = \bigcup_{i=0}^{n} S_i \).

Thus curve \( \Gamma = \bigcup_{i=0}^{n} \gamma_i \) are positively oriented and are given in a parametric kind: \( x = x(s), y = y(s), s \in [0, L] \).

Using BIEM and expression (5) we shall put problem (2)-(4) to the following system of boundary integral equations:

\[
\begin{align*}
\Gamma \kappa_i - \Gamma \kappa_i \leq C(\Gamma) \int_{0}^{x} \omega_0(x) x_0(x) x^2 dx + \\
+ \int_{0}^{x} \omega_0(x) x_0(x) x^2 dx + \left\| \Omega \right\| \int_{0}^{x} \omega_0(x) x_0(x) x^2 dx + \\
\left\| \Omega \right\| \int_{0}^{x} \omega_0(x) x_0(x) x^2 dx,
\end{align*}
\]

where \( C(\Gamma) \) is constant, depending only on \( \Omega \) – the sequence of partitions of \( \Gamma \); \( \{\Omega \} \) – the sequence of positive numbers such that the pair \( \{\Omega \}, \{\Omega \} \) satisfies the condition \( 2 \leq \left\| \Omega \right\| \leq p \).

Theorem (main)

Let

\[
\int_{0}^{x} \frac{\xi(x)}{x} dx < +\infty
\]

and let the equation (12) have the solution \( \tilde{f} \in C_f \) (the set of continuous functions on \( \Gamma \)). Then \( \exists N_0 \in \mathbb{N} = \{1, 2, \ldots \} \) such that for any \( \forall N > N_0 \) the discrete system, obtained by using the discrete double layer potential operator (its properties has been studied), has unique solution

\[
\text{where } C(\Gamma) = \left\| \Omega \right\| \int_{0}^{x} \omega_0(x) x_0(x) x^2 dx + \left\| \Omega \right\| \int_{0}^{x} \omega_0(x) x_0(x) x^2 dx.
\]

\( I_{\alpha, \Gamma} f(z) \) - two parameter (depending on \( \tau \) and \( \delta \) parameters) quadrature formula for logarithmic double layer potential; \( \tilde{f} (z) \) - double layer logarithmic potential operator; with \( C(\Gamma) \) - constant, dependent only from a curve \( \Gamma \); \( f(z) \) - a module of a continuity of functions \( f \);

\[
\left( I_{\alpha, \Gamma} f(z) \right) = \sum_{j \in \mathbb{N}} \frac{f(z_{j+1}) + f(z_{j-1})}{2}.
\]

3.2. The given technique of calculation of a temperature field of a blade can be applied to blades with the plug – in condition of interfaces between segments of a partition of an outline as equalities of temperatures and heat flows.
Were $T_k$ - unknown optimum temperature of a wall of a blade from a leg of a cooling air.

3.3. The developed technique for the numerical decision of stationary task heat conduction in cooled blades can be spread also on cvazistationary case. Let's consider a third regional task for cvazilines of the heat conduction equation:

$$\frac{\partial (T \frac{\partial T}{\partial x})}{\partial x} + \frac{\partial (T \frac{\partial T}{\partial y})}{\partial y} = 0$$

For preservation converging composed lines in a regional condition (11) we shall take advantage of substitution Kirghof:

$$A = \int \lambda(\xi) d\xi$$

Then the equation (10) is transformed to the following equation Laplas:

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = 0$$

So, the stationary task (13), (15) is decided by method of the boundary integrated equations . If the decision L (x, y) in a point (x, y) of a linear third regional task (13), (15) for the equation Laplas to substitute in (12) and after integration to decide(solve) the appropriate algebraic equation, which degree is higher than a degree of function $T$, we shall receive meaning of temperature $T$ (x, y) in the same point. Thus in radicals the algebraic equation of a degree above fourth is decided

$$a^0T^{x^4} + a^1T^{x^3} + a^2T^{x^2} + a^3T + a_4 = A,$$

That corresponds to the task $T(x,y)$ as the multinumber of a degree above third. In result the temperature field will be determined as a constant approximation, as the boundary integrated equations . If the decision L (x, y) in a point (x, y) of a linear third regional task (13), (15) for the equation Laplas to substitute in (12) and after integration to decide(solve) the appropriate algebraic equation, which degree is higher than a degree of function $T$, we shall receive meaning of temperature $T$ (x, y) in the same point. Thus in radicals the algebraic equation of a degree above fourth is decided

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$$a^0T^{x^4} + a^1T^{x^3} + a^2T^{x^2} + a^3T + a_4 = A,$$
3.6.2. Taking it to simple algebraic type:

\[ \psi = V_e \left( (y \cos \alpha_e - x \sin \alpha_e) - \frac{1}{2} \sqrt{\frac{\pi}{t}} \left( \frac{2 \pi}{t} \right) \sum_{j=1}^n \left( \sin^2 \frac{2 \pi}{t} (y - y_j) \right) \Delta y \right) \]

3.7. The data of speed distribution along the profile contour are primary for determining outer boundary heat exchange conditions. For designing local coefficient heat transfer at known geometry of the cooling scheme, for definition of the convective heat exchange local coefficients of the cooler 

\[ \text{Distribution of speed along the profile contour can be determined by solving integral equation for current function} \]

\[ \frac{\partial \psi}{\partial s} = 0, \quad s \text{ from the solution of system of linear algebraic equations.} \]

3.8. Distribution of speed along the profile contour can be determined by solving integral equation for current function [10,11]:

\[ \psi = V_e \left( (y \cos \alpha_e - x \sin \alpha_e) - \frac{1}{2} \sqrt{\frac{\pi}{t}} \left( \frac{2 \pi}{t} \right) \sum_{j=1}^n \left( \sin^2 \frac{2 \pi}{t} (y - y_j) \right) \Delta y \right) \]

taking it to simple algebraic type:

\[ \psi = V_e \left( (y \cos \alpha_e - x \sin \alpha_e) - \frac{1}{2} \sqrt{\frac{\pi}{t}} \left( \frac{2 \pi}{t} \right) \sum_{j=1}^n \left( \sin^2 \frac{2 \pi}{t} (y - y_j) \right) \Delta y \right) \]

3.9. The task of determining inner boundary heat exchange conditions is necessary. For example, for calculation of heat transfer, for determination of the convective heat exchange local coefficients of the coolant by the standard empirical formulas, is necessary to have preliminary of flow distribution in cooling channels [12,13].

For determining distribution of flow in the blade cooling system equivalent hydraulic scheme is built.

\[ \text{Cross section area} \]

\[ \Delta y \]

\[ \text{Number of typical elements connected to node} \]

\[ \text{Coefficient of hydraulic resistance} \]

\[ \text{Discharge of coolant on the element} \]

\[ \text{Pressure loss of the coolant on element} \]

\[ \text{Mean area of the cross-section passage of elements} \]

\[ \text{Coefficient of hydraulic conductivity of the circuit element} \]

\[ \text{Cooling air consumption in the spray stream through the perforation deflector holes and slot channel between the deflector and vanes wall} \]

\[ \text{Relative height of the slot channel between deflector and vane wall} \]

\[ \text{Relative width of the deflector} \]

\[ \text{Reynolds criterion in the formula} \]

\[ \text{Mean coefficient of heat transfer at the inner surface of the vane wall in the area of perforated deflector} \]

\[ \text{Coefficient of heat transfer at the inner surface of the vane wall} \]

\[ \text{Mean area of the cross-section passage of elements} \]

\[ \text{Coefficient of hydraulic resistance} \]

\[ \text{Coolant flow temperature and pressure: on the entrance to the stage} \]

\[ \text{Coolant flow in the element, and coefficient of hydraulic resistance of this element correspondingly.} \]

\[ \text{Coefficient of hydraulic conductivity of the circuit element} \]

\[ \text{Cooling tract, which includes the area of deflector perforation coefficients of hydraulic resistance in spray} \]

\[ \text{Flow rates, which were obtained while generalizing experimental data} \]

\[ \text{Use of nonlinear algebraic equations (21) is solved by Zeidel method with acceleration, taken from:} \]

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\[ \text{Coefficient of hydraulic resistance of this element} \]

\[ \text{Flow rates, which were obtained while generalizing experimental data} \]

\[ \text{Coolant flow vector angle} \]

\[ \text{Coolant flow in the element, and coefficient of hydraulic resistance of this element correspondingly.} \]

\[ \text{Flow speeds, which were obtained while generalizing experimental data} \]

\[ \text{Coolant flow vector angle} \]

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The relative gas speed on the exit from the cascade: 
\[ \rho_{\text{at}} = 0.75 \times 10^6 \text{ Pa} \].

The geometrical model of the nozzle blades (fig.3), diagrams of speed distributions \( V \) and convective heat exchange local coefficients of gas along profile contour (fig.4) are received.

The geometrical model (fig.5) and the cooling tract equivalent hydraulic scheme (fig.6) are developed. Cooler basics parameters in the cooling system and temperature field of blade cross section (fig.7) are determined.

5. CONCLUSIONS

The reliability of the methods was proved by experimental investigations of blades heat and hydraulic characteristics in "Turbine construction" laboratory (St. Petersburg, Russia). Geometric model, equivalent hydraulic schemes of cooling tracts have been obtained, cooler parameters and temperature field of "Turbo machinery Plant" enterprise gas turbine nozzle blade of the 1st stage have been determined (Yekaterinburg, Russia). Methods have demonstrated high efficiency at repeated and polivariant calculations, on the basis of which the way of blade cooling system modernization has been offered.

The application of perfect methods of calculation of temperature fields of elements of gas turbines is one from actual problems of an air engine building. The efficiency of these methods in the total renders direct influence to operational manufacturability and reliability of elements of designs, and also on acceleration characteristics of the engine.

REFERENCES


APPENDIX

Correction algorithm

Input signals \( X \)

Target signals \( Y \)

Deviations

Training quality

Random-number generator

NN

Parameters

Fig.1. System for network-parameter (weights, threshold) training (with feedback)

Fig.2. Neural network structure for multiple linear regression equation
Fig. 3. The cascade of profiles of the nozzle cooled blade.

Fig. 4. Distribution of the relative speeds \( \frac{V_1}{\lambda} \) and of gas convective heat exchange coefficients \( \alpha_g \) along the periphery of the profile contour.

Fig. 5. Geometrical model with foliation of design points of contour (1-78) and equivalent hydraulic schemes reference sections (1-50) of the experimental nozzle blade.

Fig. 6. The equivalent hydraulic scheme of experimental nozzle blade cooling system.

Fig. 7. Distribution of temperature along outside (▲) and internal (■) contours of the cooled nozzle blade.