Reliability Evaluation of Composite Electric Power System Based On Latin Hypercube Sampling

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Abstract—This paper investigates the suitability of Latin Hypercube sampling (LHS) for composite electric power system reliability analysis. Each sample generated in LHS is mapped into an equivalent system state and used for evaluating the annualized system and load point indices. DC loadflow based state evaluation model is solved for each sampled contingency state. The indices evaluated are loss of load probability, loss of load expectation, expected demand not served and expected energy not supplied. The application of the LHS is illustrated through case studies carried out using RBTS and IEEE-RTS test systems. Results obtained are compared with non-sequential Monte Carlo simulation and state enumeration analytical approaches. An error analysis is also carried out to check the LHS method’s ability to capture the distributions of the reliability indices. It is found that LHS approach estimates indices nearer to actual value and gives tighter bounds of indices than non-sequential Monte Carlo simulation.

Keywords—Composite power system, Latin Hypercube sampling, Monte Carlo simulation, Reliability evaluation, Variance analysis.

I. INTRODUCTION

Composite system reliability evaluation is concerned with the problem of determining the adequacy of the combined generation and transmission system in regard to providing a dependable and suitable supply at the load points. The indices evaluated such as loss of load probability (LOLP), loss of load expectation (LOLE), expected demand not supplied (EDNS) and expected energy not supplied (EENS) are very useful parameters to help the planning engineers to make decisions. The methods employed for evaluating the indices can be categorized into analytical and simulation methods. Accurate evaluation of indices requires complete investigation and analysis of each possible outage condition of the system. Analytical methods are based on enumeration of states which are seen to be more efficient when a relatively small number of states accounts for most of the probability of the state space. This situation is typical in pure transmission reliability studies, in which the line outage probabilities are usually low and as a result independent combination of several line outage occurrences are unlikely and can be eliminated. But in composite system with large number of components the states enumerated are more and leads to large computational requirement [1].

Simulation methods which are based on Monte Carlo techniques are performing better with more number of outage combinations. This situation more often occur in composite reliability evaluation, in which higher level outages are more likely due to the relatively higher forced outage rate of generating units. Monte Carlo simulation (MCS) methods estimate the indices by simulating the actual random failure behavior of the system [2]. Monte Carlo sampling for reliability evaluation can be classified into sequential and non-sequential sampling. The sequential sampling simulates the chronological behavior of the system operation which requires more computational effort than non-sequential sampling and analytical methods. Non-sequential sampling has high computational efficiency but cannot simulate the chronological aspects of system operation. The major limitation of this method is that number of states sampled increases with the required indices accuracy. The required number of sample depends on the variance of the random variable and the desired accuracy. A smaller variance of the estimate can reduce the number of samples which can be achieved by employing a suitable variance reduction mechanism [3].

Variance reduction techniques such as stratified sampling [4], importance sampling [5], control variates [6] and antithetic variates [7] have been proposed to reduce the computational burden of MCS. The main problem with these techniques is their suitability for different system configurations. This means that one technique is effective for a particular system network and has no effect on the other system. Latin Hypercube sampling (LHS), which has many desirable features of stratified sampling and random sampling has been successfully applied to generation system reliability evaluation. Discrete version of LHS is also presented for reliability evaluation. This analysis is performed with fixed sample size and results of LHS approach and simple MCS approach are compared with the results of state enumeration approach. The comparison shows LHS gives better estimate than MCS with same sample size [8]. Panida Jurutitijaroen and Zhen Shu applied LHS to the reliability analysis of power system which includes renewable energy sources with an emphasis on the fluctuation of bus loads and intermittent behavior of renewable generations such as wind and solar power [9].

The main objective of this paper is to investigate the suitability of LHS for composite electric power system reliability analysis. An error analysis is also carried out to check the LHS based method’s ability to capture the distributions of the reliability indices. Case studies on the
RBTS (Roy Billington test system) and the IEEE-RTS (Reliability test system) systems are presented and the results are compared with non-sequential MCS and state enumeration analytical approaches.

The rest of the paper is organized as follows. Section II gives a brief explanation about the variance reduction techniques employed in composite system reliability evaluation. Section III introduces a dc loadflow based system state evaluation model which is solved for each contingency state to determine loss of load curtailment necessary for maintaining real power balance. Section IV presents the basis of LHS and its implementation for composite system reliability analysis. Section V presents the numerical results of proposed LHS approach to the standard test systems. Finally the conclusions are presented in Section VI.

II. VARIANCE REDUCTION TECHNIQUES

The problem of calculating the reliability indices in MCS is equivalent to calculating the expected value of a test function

$$E(F) = \sum_{x \in X} F(x)P(x)$$

where \(F(x)\) is the test function and \(P(x)\) is the probability of that state.

The estimate of the expected value of the test function with \(n_s\) samples is given by

$$\hat{E}(F) = \frac{1}{n_s} \sum_{j=1}^{n_s} F(x_j)$$

The uncertainty around the estimate \(\hat{E}(F)\) is given by the variance of the estimate

$$V(\hat{E}(F)) = V(F)/n_s$$

where \(V(F)\) is the variance of the test function.

The co-efficient of variation \(\beta\) represents the uncertainty of the estimate (otherwise accuracy of the estimate)

$$\beta = \sqrt{V(\hat{E}(F))} / \hat{E}(F)$$

For better estimate of \(\hat{E}(F)\) the value of \(\beta\) should be small and the relation between number of samples for desired \(\beta\) is given by

$$n_s = V(F)(\beta \hat{E}(F))^2$$

The above equation indicates that to achieve the desired accuracy \(\beta\) with less number of samples, the variance of the test function should be reduced. Some of the variance reduction techniques employed for composite system reliability analysis was stratified sampling, important sampling, control variates and antithetic variates. Stratified sampling consists of stratifying the sample space and then constructing the estimates from each stratum. The sample space is stratified by simply choosing a partition of the input parameter space. In Important sampling, the sampling distribution is distorted in such a manner to produce a estimate with a lower variance by sampling more in the important regions. Control variates attempts to take the advantage of correlation between certain random variables for obtaining a variance reduction. Antithetic variates try to induce negative correlation by using complementary random numbers to drive the two simulation runs in a pair.

LHS incorporates many of the desirable features of stratified sampling and random sampling and also produces more stable analysis outcome than random sampling. LHS is a probabilistic procedure that each sample element can be associated with a weight.

III. SYSTEM STATE EVALUATION MODEL

State evaluation is an essential step in composite system reliability assessment. For each contingency state sampled in any of the simulation approach, the dc load flow based load curtailment model is used to examine the adequacy of the system by rescheduling generation outputs in order to maintain real power balance and alleviate line overloads and at the same time, to avoid load curtailment if possible or to minimize total load curtailment if unavoidable. If real power balance is achieved without load curtailment then the state belongs success state otherwise it belongs to failure state and load curtailment necessary to attain real power balance is calculated by solving the following model.

$$\text{Min } C_l = \sum_{i\in NC} (w_i \sum_{j=1}^{l_i} a_{ij} C_{lj})$$

Subject to

$$P_{line_o} = \sum_{k=1}^{N} A_{ok} \left( P_{g_k} + \sum_{j=1}^{l_i} C_{kj} - P_{d_k} \right)$$

$$\text{( } o = 1, \ldots, L \text{ )}$$

$$\sum_{i\in NC} P_{g_i} + \sum_{i\in NC} (\sum_{j=1}^{l_i} C_{ij}) = \sum_{i\in NC} P_{d_i}$$

$$P_{g_{i,\text{min}}} \leq P_{g_i} \leq P_{g_{i,\text{max}}} \text{ ( } i = 1, \ldots, NG \text{ )}$$

$$0 \leq C_{lj} \leq \gamma_j P_{d_i} \text{ ( } i \in NC; j = 1, \ldots, m \text{ )}$$

$$\| P_{line_o} \| \leq \| P_{line_o^{\text{max}}} \| \text{ ( } o = 1, \ldots, L \text{ )}$$

where \(C_{lj}\) is the \(j^{th}\) load curtailment sub variable at bus \(i\), \(P_{g_i}\) is the generation at bus \(i\), \(P_{d_i}\) is the load demand at bus \(i\), \(P_{line_o}\) is the line flow of line \(o\), \(P_{g_{i,\text{min}}}\) is the minimum generation at bus \(i\), \(P_{g_{i,\text{max}}}\) is the maximum generation at bus \(i\), \(P_{line_o^{\text{max}}}\) is the maximum value of line flow of line \(o\), \(A_{ok}\) is the element of the relation matrix between line flows and power injection, \(NC\) is the sets of all load buses, \(NG\) is the sets of all generator buses, \(L\) is the number of lines, \(N\) is the number of buses, \(l_i\) is the number of load curtailment subvariables at bus \(i\), \(a_{ij}\) the weighting factor corresponding to subvariable \(j\), \(\gamma_j\) is the load percentage associated with each
subvariable j, W_i is the weighting factor corresponding to each load bus.

IV. LATIN HYPERCUBE SAMPLING APPROACH

LHS can be applied to multiple variables and viewed as an ‘n’ dimensional extension of Latin square sampling. It can be incorporated into an existing Monte Carlo model very easily and work with variables following any analytical probability distribution. It emphasizes uniform sampling of the univariate distributions. LHS accomplishes this by stratifying the cumulative distribution function and randomly sampling within the strata. Uniform sampling increases the realization efficiency while randomizing within the strata prevents the introduction of a bias and avoids the extreme value effect associated with simple stratified sampling [10]-[12].

A. Generation and Variance of Samples

Let ‘ns’ be the sample size and ‘n’ be the number of variables in x=(x_1,x_2,…,x_n) of function y=F(x), where the variables of x are independent. The range of each variable is partitioned into ‘ns’ non-overlapping intervals on the basis of equal probability size 1/ns, one value from each interval is selected at random with respect to the probability density in that interval. The ‘ns’ values thus obtained for x_1 are paired in a random manner with ‘ns’ values of x_2. These ‘ns’ pairs are combined with values of x_3 and so on, until ‘ns’ samples of ‘n’ variables are formed. The total sample space contains n^s cells out which ‘ns’ samples are picked in the above manner.

For a test function F(x) with expected value E(F) and estimate of expected value \( \bar{E}(F) \), the variance of the estimate in LHS is given by

\[
V(\bar{E}(F)) = \frac{1}{ns} V(F) + \frac{ns-1}{ns} Cov( F(x_1), F(x_2), ..., F(x_n) ) \quad (12)
\]

From the above relation it is found that variance of the estimate in LHS decreases when the covariance term \( Cov( F(x_1), F(x_2), ..., F(x_n) ) \) is negative.

The sample size fixed in LHS for composite system reliability evaluation is in the order of several thousands, by virtue of the fact (ns-1)/ns is almost equal to one. This establishing the sufficiency of the sample fixed and approaches ‘ns’ to infinity. As per the proof given by Michael Stein [13] when ns \( \rightarrow \infty \), the covariance term is asymptotically non positive for ‘n’ independent variables. Comparison of (3) and (12) shows that the variance achieved in LHS is smaller than MCS of same sample size. This shows that LHS gives better estimate with smaller co-efficient of variation \( \beta \) than MCS of the same sample size.

B. Implementation of LHS for Composite System Reliability Analysis

The generating units and transmission lines of the system are represented by a two state markov model with failure rate \( \lambda \) and repair rate \( \mu \). The probability of the system component ‘i’ in down state and up state is given by

\[
P_t^d = \frac{\lambda_i}{\lambda_i + \mu_i} \quad (13)
\]

\[
P_t^u = \frac{\mu_i}{\lambda_i + \mu_i} \quad (14)
\]

The LHS algorithm for composite system reliability analysis is summarized as follows

1. Set the number of samples equal to ‘ns’ and number of components or variables equal to ‘n’.
2. Compute the down state and upstate probabilities for each component ‘i’ using (13) and (14).
3. Partition the cumulative probability distribution of each component ‘i’ into ‘ns’ numbers of non-overlapping intervals each with probability ‘1/ns’ and randomly select a single value from each sub interval i.e. for each component ‘ns’ values are picked.
4. The ‘ns’ values obtained for component 1 are randomly paired with the ‘ns’ values of component 2. The ‘ns’ pairs are combined in a random manner with the ‘ns’ values of components 3 to form ‘ns’ triplets and so on, until a set of ‘ns’ number of n-tuples are formed. This set of n-tuples form LHS samples.
5. Convert the LHS samples into equivalent system states. For component ‘i’ of sample ‘j’ if the value of component ‘i’ lies in the interval of 0 to \( P_t^d \) then the component is in down state, otherwise it is in upstate.
6. Calculate the test functions for each system state. For each contingency state, a system state evaluation model given in Section III is solved to classify the state as failure or success and based on that the value of test functions are calculated for that sample. The value of test function F(x) for LOLP index is equal to one if x is a failure state otherwise it is zero. For the index EENS the test function F(x) represents the amount of load curtailment required to alleviate the operating constraint violations and maintain power balance. In that case F(x)=0 if there is load curtailment associated to a failure state x; Otherwise F(x)= \( \beta \) indicates that x is a success state. The values of test functions for all samples are calculated and the estimate of the reliability indices is evaluated based on (2).

V. NUMERICAL RESULTS

The proposed approach for composite system reliability evaluation has been implemented on RBTS [14] and IEEE--RTS [15] systems. Comparison with the MCS approach has carried out by setting the sample size of LHS equal to the samples required for achieving co-efficient of variation \( \beta \) in non-sequential MCS. It is found from Section IV A, LHS gives tighter bounds on estimated indices than simple random MCS for same number of sample size [8], [13]. A comparative study on analytical method and MCS method were done to verify the performance of the proposed approach. Only peak load levels were used for the purpose of this study. The proposed approach has been implemented in matlab software. An error analysis is carried out by calculating the percentage error of the estimates which is found from averaging the
percentage absolute deviation from the actual value over all runs of sample.

\[
\text{Percentage Error} = \frac{1}{nr} \sum_{z=1}^{nr} \left( \frac{\text{Estimated}_z - \text{Actual}}{\text{Actual}} \right) \times 100 \quad (15)
\]

where \(\text{Estimated}_z\) is the reliability index estimated in run ‘\(z\)’ in either LHS or MCS approach, \(\text{Actual}\) is the reliability index evaluated in state enumeration analytical method and \(nr\) is the number of runs or trials.

### A. Case 1: RBTS Test System

The RBTS system consists of 6 buses, 9 transmission lines and 11 generators. The minimum and maximum ratings of the generation units are 5 MW and 40 MW respectively. The total peak load for the system is 185 MW while the total generating capacity is 240 MW. The pre-selected sample size is set to 50000 for LHS, which is used by Jonnavithula [18] for estimating indices in non-sequential MCS. The total number of components in the system is 20. For each component, the cumulative probability is divided into 50,000 non-overlapping intervals with equal probability of 0.00002. The total LHS sample space for 50,000 samples is \((50,000)^{20}\) and out of which 50,000 samples are generated using the procedure given in Section IV.

#### TABLE I

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>LOLP</th>
<th>LOLE hr/yr</th>
<th>EDNS MW</th>
<th>EENS MWhr/yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.00095</td>
<td>8.29920</td>
<td>0.00416</td>
<td>36.34176</td>
</tr>
<tr>
<td>3</td>
<td>0.00502</td>
<td>43.85472</td>
<td>0.06031</td>
<td>526.86820</td>
</tr>
<tr>
<td>4</td>
<td>0.00593</td>
<td>51.84048</td>
<td>0.03562</td>
<td>311.17630</td>
</tr>
<tr>
<td>5</td>
<td>0.00049</td>
<td>4.28064</td>
<td>0.00159</td>
<td>13.89024</td>
</tr>
<tr>
<td>6</td>
<td>0.00197</td>
<td>17.20992</td>
<td>0.02102</td>
<td>183.63070</td>
</tr>
</tbody>
</table>

The annualized load point indices evaluated by this approach are shown in Table I. Individual load point indices are necessary for identifying the weak points in the system and help the reliability engineers in planning the optimum response of equipment investment.

#### TABLE II

<table>
<thead>
<tr>
<th>Approach</th>
<th>LOLP</th>
<th>LOLE hr/yr</th>
<th>EDNS MW</th>
<th>EENS MWhr/yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHS</td>
<td>0.01002</td>
<td>87.5347</td>
<td>0.1227</td>
<td>1071.907</td>
</tr>
<tr>
<td>MCS-state sampling [18]</td>
<td>0.01014</td>
<td>88.5830</td>
<td>0.1239</td>
<td>1082.630</td>
</tr>
<tr>
<td>Analytical [17]</td>
<td>0.00976</td>
<td>85.2634</td>
<td>0.1201</td>
<td>1052.300</td>
</tr>
</tbody>
</table>

The annualized system indices evaluated by LHS approach are presented in Table II. It also gives the comparison of the evaluated indices with the results of analytical [17] and non-sequential state sampling MCS [18] based approaches. It is found that indices estimated in LHS approach are in close agreement with the existing approaches and are nearer to analytical method than MCS approach. The accumulation behavior of EENS index is shown in Fig. 1.

### B. Case 2: IEEE-RTS Test System

IEEE–RTS system consists of 24 buses, 38 transmission lines and 32 generators with 10 of the buses connected to generators. The total peak load for the system is 2250 MW while the total generating capacity is 3405 MW. Only peak load levels were used for the purpose of this study. The pre-selected sample size is 10000 for LHS, which is used by Jonnavithula [18] for estimating indices in non-sequential MCS. The total number of components in the system is 70. For each component, the cumulative probability is divided into 10000 non-overlapping intervals with equal probability of 0.0001. The total sample space for 10000 samples consists of \((10000)^{70}\) and out of which 10000 samples are generated using the procedure given in Section IV.

#### TABLE III

<table>
<thead>
<tr>
<th>Sample size</th>
<th>LHS</th>
<th>MCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOLP</td>
<td>EENS MWhr/yr</td>
<td>LOLE hr/yr</td>
</tr>
<tr>
<td>10000</td>
<td>1071.907</td>
<td>87.5347</td>
</tr>
<tr>
<td>20000</td>
<td>1082.630</td>
<td>88.5830</td>
</tr>
<tr>
<td>50000</td>
<td>1052.300</td>
<td>85.2634</td>
</tr>
</tbody>
</table>

The percentage error in LOLP and EENS estimates are presented in Table III. The estimates are closer to the actual values when the sample size increases in both MCS and LHS approaches. The error of LOLP index for 10000 samples is 13.21% and its estimate is nearer to actual value for 50000 samples, where error is only 2.66%. The error in EENS index for 10000 samples is 5.23% and its estimate is nearer to actual value for 50000 samples where error is only 1.86%. This reflects the LHS ability of predicting the probability distributions of indices accurately. LHS estimates LOLP and EENS better than MCS approach with same sample size.

The performance of the proposed LHS approach has been tested by carrying an error analysis with non-sequential state sampling MCS with fixed sample sizes of 10000, 20000 and 50000 with 10 runs of simulation. The percentage error is calculated for LOLP and EENS indices using (15) for the above selected sample sizes.

#### TABLE III

<table>
<thead>
<tr>
<th>PERCENTAGE ERROR OF ESTIMATED LOLP &amp; EENS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample size</td>
</tr>
<tr>
<td>10000</td>
</tr>
<tr>
<td>20000</td>
</tr>
<tr>
<td>50000</td>
</tr>
</tbody>
</table>

The percentage error in LOLP and EENS estimates are closer to the actual values when the sample size increases in both MCS and LHS approaches. The error of LOLP index for 10000 samples is 13.21% and its estimate is nearer to actual value for 50000 samples, where error is only 2.66%. The error in EENS index for 10000 samples is 5.23% and its estimate is nearer to actual value for 50000 samples where error is only 1.86%. This reflects the LHS ability of predicting the probability distributions of indices accurately. LHS estimates LOLP and EENS better than MCS approach with same sample size.
The annualized load point indices evaluated for all 17 load points by this approach are given in Table IV. It is found that load point 18 has highest probability of failure with 61823.08 MWhr of expected energy not supplied per year. The annualized system indices evaluated in this approach with preselected sample size of 10,000 are shown in Table V. The results are compared with the results of analytical [16] and non-sequential MCS [18] approaches. It is found from the table that indices estimated in the proposed LHS approach are in close agreement with MCS and to analytical method. The accumulation behavior of EENS index is given in Fig. 2.

The performance of the proposed LHS approach for IEEE-RTS system with fixed sample sizes of 2000, 5000 and 10000 has been verified by carrying an error analysis with non-sequential MCS. The indices considered in the error analysis are LOLP and EENS.

The percentage error of LOLP and EENS estimates are presented in Table VI. It is found from table the estimates are closer to the actual values when the sample size increases in both MCS and LHS approaches. The error of LOLP index for 2000 samples is 15.57% and its estimate is nearer to actual value for 10000 samples, where error is only 4.64%. The error in EENS index for 2000 samples is 14.39% and for 10000 samples error is only 8.3%. This reflects tighter bounds of indices are achieved in LHS method than MCS method.

The indices estimated in simulation approaches are usually represent the mean values of the distributions of the indices. The mean values are generally dominated by the high probability region of the distribution. It is found from Tables II & V, both LHS and MCS approaches estimate the indices nearer to actual values. The mean value is close to the actual value when the sample size increases in both MCS and LHS approaches. The error analysis indicate that the indices estimated are in the high probability region and from Tables III & VI, LHS gives tighter bounds of indices in comparison to MCS for the same sample size. It is inferred from the results LHS achieve superior estimate of reliability indices than random non-sequential MCS approach.

VI. CONCLUSION

This paper presents the application of LHS for evaluating the composite system reliability indices. The main objective in simulation approach is to reduce the variance of the estimate to achieve better evaluation of reliability indices. The variance relation given in Section IV A proves that the LHS achieve smaller variance than simple MCS of same sample size. The reduction in variance of the estimate results the improvement in co-efficient variation which leads to accurate estimate of indices. A dc load flow based system state evaluation model was used for evaluating the test functions of the sampled contingency states. The applicability of the LHS approach has been verified by applying the proposed method to standard RBTS and IEEE-RTS systems. The obtained results reflect that the indices estimated are similar to existing methods. It is also found that the indices estimated in LHS approach are nearer to the benchmark analytical method which requires more computational effort. An error analysis is also carried out to check the bounds of the estimated indices. The results show LHS achieve less error in predicting the mean values of indices distribution than MCS method. This proves LHS ability to give tighter bounds of indices than MCS and leads to...
superior results than existing random non-sequential MCS. The proposed approach is simple and easy to adopt for sampling the entire region of probability distribution of individual components.

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REFERENCES


