Abstract—In this paper we propose an algorithm based on higher order cumulants, for blind impulse response identification of frequency radio channels and downlink (MC–CDMA) system Equalization. In order to test its efficiency, we have compared with another algorithm proposed in the literature, for that we considered on theoretical channel as the Proakis’s ‘B’ channel and practical frequency selective fading channel, called Broadband Radio Access Network (BRAN C), normalized for (MC–CDMA) systems, excited by non-Gaussian sequences. In the part of (MC–CDMA), we use the Minimum Mean Square Error (MMSE) equalizer after the channel identification to correct the channel’s distortion. The simulation results, in noisy environment and for different signal to noise ratio (SNR), are presented to illustrate the accuracy of the proposed algorithm.

Keywords—Blind identification and equalization, Higher Order Cumulants, (MC–CDMA) system, MMSE equalizer.

I. INTRODUCTION

In the literature several works show that the signal processing techniques using Higher-Order Statistics (HOS) or cumulants have attracted considerable attention [1,2,3,4,5,6,10]. Considerable work has been done in the area of model parameters identification [8], which consist in using second order statistics. But, these statistics are sensible to additive Gaussian noise. Thus, their performances degrade when the output is noisy and they are incapable to identify the nonminimum phase systems [2,7]. In this work, we propose on blind algorithm based on higher order cumulants, this approach allows the resolution of the insoluble problems using the second order statistics. In order to test the efficiency of the proposed algorithm we have compared with the Zhang et al. Algorithm [3]. In this paper we have considered on theoretical channel as the Proakis’s ‘B’ channel, and practical frequency selective fading channel called Broadband Radio Access Network (BRAN C) [11,12] normalized for MC-CDMA systems, exited by a non-Gaussian sequences, for different propagation paths. The problem encountered in communication is the synchronization between the transmitter and the receiver, due to the echoes and reflection between the transmitter and the receiver. Synchronization errors cause loss of orthogonality among sub-carriers and considerably degrade the performance especially when large number of subcarriers presents [14]. In this paper, we propose a blind identification algorithm based on higher order cumulants, for identification of the Broadband Radio Access Network Channel such as BRAN C, compared with the Zhang et al algorithm. The application of this algorithms in the context of downlink MC-CDMA equalization is also considered.

II. PROBLEM FORMULATION

We consider the following discrete time, causal, linear of the Finite Impulse Response (FIR) system represented on figure 1 and described by equations (1) and (2), with the following assumptions : In order to simplify the construction of the algorithm we assume that:

- The input sequence, \( x(k) \), is independent and identically distributed (i.i.d) zero mean, and non-Gaussian.
- The system is causal and truncated, i.e. \( h(k) = 0 \) for \( k < 0 \) and \( k > q \), where \( h(0) = 1 \).
- The system order \( q \) is known.
- The measurement noise sequence \( n(k) \) is assumed zero mean, i.i.d, Gaussian and independent of \( x(k) \) with unknown variance.

The problem statement is to identify the parameters of the system \( h(k)_{k=1,q} \) using the cumulants of the measured output process \( y(k) \).

The output time series is described by

\[
x(k) \xrightarrow{\text{BRAN Channel (NMP)}} y(k) + n(k)
\]

Fig. 1. Channel model
\[ y(k) = \sum_{i=1}^{q} x(i)h(k - i). \]  

(1)

With noise:

\[ r(k) = y(k) + n(k), \]

(2)

where \( n(k) \) is the noise sequence.

### III. PROPOSED ALGORITHM: ALGO–ZSS

The equation proposed in [8] presents the relationship between different \( m^{th} \) and \( n^{th} \) cumulants of the output signal, \( y(n) \), as follows:

\[
\sum_{j=0}^{q} h(j)[\prod_{k=1}^{m-1} h(j + \tau_k)]C_{nq}(\beta_1, \ldots, \beta_{m-1}, j + \alpha_1, \ldots, j + \alpha) = \varepsilon_{n,m} \sum_{i=1}^{q} h(i)[\prod_{k=1}^{n-1} h(i + \beta_k)]C_{mq}(\tau_1, \ldots, \tau_{n-1}, i + \alpha_1, \ldots, i + \alpha),
\]

(3)

with \( \varepsilon_{n,m} = \frac{\varepsilon_{mn}}{\varepsilon_{nm}} \) and \( 1 \leq s \leq \min(m, n) - 2 \), where \( \varepsilon_{mn} \) represents the \( n^{th} \) order cumulants of the excitation signal \( x(i) \) at origin.

Based on the relationship (3) we can develop the following algorithm based on the Higher Order Cumulants (HOC).

If we take \( n = 4 \) and \( m = 3 \) into (3) we obtain:

\[
\sum_{j=0}^{q} h(j)h(j + \tau_1)C_{4q}(\beta_1, \beta_2, j + \alpha_1) = \frac{\varepsilon_{8,6}}{\varepsilon_{6,8}} \sum_{i=0}^{q} h(i)h(i + \beta_1)h(i + \beta_2)C_{3q}(\tau_1, i + \alpha_1).
\]

(4)

If we take \( \beta_1 = \beta_2 = 0 \) into (4) we obtain the following equation:

\[
\sum_{i=0}^{q} h^3(i)C_{3q}(\tau_1, i + \alpha_1) = \frac{\varepsilon_{8,6}}{\varepsilon_{6,8}} \sum_{j=0}^{q} h(j)h(j + \tau_1)C_{4q}(0, 0, j + \alpha_1).
\]

(5)

Else if we take \( \tau_1 = q \) into (5). The considered system is causal we obtain the Following equation:

\[
\sum_{i=0}^{q} h^3(i)C_{3q}(q, i + \alpha_1) = \frac{\varepsilon_{8,6}}{\varepsilon_{6,8}} h(0)h(q)C_{4q}(0, 0, \alpha_1),
\]

where \( h(0) = 1 \).

\[
\sum_{i=1}^{q} h^3(i)C_{3q}(q, i + \alpha_1) = \frac{\varepsilon_{8,6}}{\varepsilon_{6,8}} h(q)C_{4q}(0, 0, \alpha_1) - C_{3q}(q, \alpha_1).
\]

(7)

To simplify the (7), we consider the relation of Brillinger and Rosenblatt already used in [7,9,10] describe with following equation for \( m = 4 \):

\[
C_{4q}(t_1, t_2, t_3) = \frac{\varepsilon_{8,6}}{\varepsilon_{6,8}} \sum_{i=0}^{q} h(i)h(i + t_1)h(i + t_2)h(i + t_3).
\]

(8)

If \( t_1 = t_2 = t_3 = q \) Eq. (8) becomes:

\[
C_{4q}(q, q, q) = \frac{\varepsilon_{8,6}}{\varepsilon_{6,8}} h^3(q).
\]

(9)

Else if \( t_1 = t_2 = q \) and \( t_3 = 0 \) (8) reduces:

\[
C_{4q}(q, q, 0) = \frac{\varepsilon_{8,6}}{\varepsilon_{6,8}} h^2(q).
\]

(10)

From (9), (10) we obtain:

\[ h(q) = \frac{C_{4q}(q, q, q)}{C_{4q}(q, q, 0)}. \]

(11)

Thus, we based on (11) for eliminating \( h(q) \) in (7), we obtain the following equation:

\[
\sum_{i=1}^{q} h^3(i)C_{3q}(i, i + \alpha_1) = \frac{\varepsilon_{8,6}}{\varepsilon_{6,8}} C_{4q}(q, q, q) C_{4q}(0, 0, \alpha_1) - C_{3q}(q, \alpha_1),
\]

(12)

with

\[-q \leq \alpha_1 \leq q.\]

(13)

Then, from (12) and (13) the system of equations can be written in matrix form as:

\[
\begin{bmatrix}
C_{3q}(q, q + 1) & \cdots & C_{3q}(q, 0) \\
C_{3q}(q, q + 2) & \cdots & C_{3q}(q, 1) \\
\vdots & \ddots & \vdots \\
C_{3q}(q, q + 1) & \cdots & C_{3q}(q, 2q)
\end{bmatrix} \times
\begin{bmatrix}
h^3(1) \\
h^3(2) \\
h^3(q)
\end{bmatrix} =
\begin{bmatrix}
\mu C_{4q}(0, 0, -q) - C_{3q}(q, -q) \\
\mu C_{4q}(0, 0, -q + 1) - C_{3q}(q, -q + 1) \\
\vdots \\
\mu C_{4q}(0, 0, 0) - C_{3q}(q, 0)
\end{bmatrix}
\]

(14)

where \( \mu = \frac{\varepsilon_{8,6}}{\varepsilon_{6,8}} C_{4q}(q, q, q) \).

or in more compact form, (14) can be written as follows:

\[ M \hat{\theta} = A. \]

(15)

The Least Squares (LS) solution of the system of (15), will be written under the following form

\[ \hat{\theta} = (M^T M)^{-1} M^T A. \]

(16)

The parameters \( h(j) \) for \( j = 1, \ldots, q \) are estimated from the estimated values \( \hat{\theta}(j) \) using the following equation:

\[ \hat{h}(j) = \sqrt{\hat{\theta}(j)}. \]

(17)

IV. ZHANG et al. ALGORITHM: ALGO–ZHANG

Zhang et al. [3] demonstrates that the coefficients \( h(j) \) for an FTR system can be obtained by the following equation:

\[
\sum_{i=0}^{q} h(i)C_{mq}^{m-1}(i-t, \ldots, 0) = C_{mq}(t, 0, \ldots, 0)C_{mq}(t, \ldots, 0)C_{mq}(q, \ldots, 0).
\]

(18)

For \( n = 4 \), from (18), we obtain the following equation:

\[
\sum_{i=0}^{q} h(i)C_{4q}^{3}(i-t, q, 0) = C_{4q}(t, 0, 0)C_{4q}(q, 0, 0)C_{4q}(q, q, 0).
\]

(19)
for \( t = -q, -q + 1, \ldots, q \)
\[
\begin{pmatrix}
C_{4y}(1 + q, q, 0) & \ldots & C_{4y}(2q, q, 0) \\
\vdots & \ddots & \vdots \\
C_{4y}(q, q, 0) & \ldots & C_{4y}(0, q, 0)
\end{pmatrix}
\]

Then, (20) can be written as follows:
\[
Mh = d,
\]
where \( M \) is the matrix of size \((2q+1) \times (q)\) elements, \( h \) is a column vector constituted by the unknown impulse response parameters \( h(k) = k = 1, \ldots, q \) and \( d \) is a column vector of size \((2q+1)\). The least squares (LS) solution of the system of (21), permits blindly identification of the parameters \( h(k) \) and without any information of the input selective channel. So, the solution will be written under the following form
\[
\hat{h} = (M^T M)^{-1} M^T d.
\]

V. APPLICATION OF MC–CDMA SYSTEM

The principle of (MC–CDMA) is to transmit a data symbol of a user simultaneously on several narrowband sub-channels. These sub-channels are multiplied by the chips of the user-specific spreading code, as illustrated in Fig. 2.

\[
\begin{align*}
S_j(t) &= \frac{1}{\sqrt{N_p}} \sum_{k=0}^{N_t-1} d_j c_{j,k} e^{2\pi f_k t},
\end{align*}
\]

where \( f_k = f_0 + \frac{k}{T_s} \), \( N_u \) is the user number and \( N_p \) is the number of subcarriers, and we consider \( L_s = N_p \).

Fig. 3 explains the principle of the transmitter for downlink (MC–CDMA) systems. We assumed that the channel is time invariant and it’s impulse response is characterized by \( P \) paths of magnitudes \( \beta_p \) and phases \( \theta_p \), the impulse response is given by the following equation
\[
h(\tau) = \sum_{p=0}^{P-1} \beta_p e^{j\theta_p} \delta(\tau - \tau_p).
\]

B. MC–CDMA Receiver

The relationship between the emitted signal \( S(t) \) and the received signal \( r(t) \) is given by:
\[
r(t) = h(t) * S(t) + n(t),
\]
where \( n(t) \) is an additive white Gaussian noise.

The downlink received MC-CDMA signal at the input receiver is given by the following equation
\[
r(t) = \sum_{p=0}^{P-1} \beta_p e^{j\theta_p} S(t - \tau_p) + n(t).
\]

In fig. 4 we represent the receiver for downlink MC-CDMA systems.

In the reception, we demodulate the signal according the code of the user. A receiver with single-user detection of the data symbols of user \( j \) is shown in fig. 5.

After the equalization and the despreading operation, the estimation \( \hat{d}_j \) of the emitted user symbol \( d_j \), of the \( j \)th user can be written by the following equation:
We pose, $h_k = a + ib$ and $g_k = c + id$. The minimization of the function $E[|e|^2]$, gives us the optimal values of $c$ end $d$.

$$c = \frac{2aE[|S_k|^2]}{2(a^2 + b^2)E[|S_k|^2] + 2E[|n_k|^2]} \quad (31)$$

$$d = \frac{-2bE[|S_k|^2]}{2(a^2 + b^2)E[|S_k|^2] + 2E[|n_k|^2]} \quad (32)$$

The minimization of the mean square error criterion, of each subcarrier as:

$$g_k = \frac{a - ib}{(a^2 + b^2) + E[|n_k|^2]} \quad (33)$$

$$g_k = \frac{h^*}{|h|^2 + \frac{1}{\varsigma_k}} \quad (34)$$

where $\varsigma_k = \frac{E[|S_k|^2]}{E[|n_k|^2]}$ with $E[|h_k|^2] = 1$

The estimated received symbol, $\hat{d}_j$ of symbol $d_j$ of the user $j$ is described by

$$\hat{d}_j = \sum_{k=0}^{N_p-1} c_{j,k} h_k^* d_j + \sum_{k=0}^{N_p-1} c_{j,k} g_k n_k. \quad (35)$$

If we assumed that the spreading code are orthogonal, i.e.,

$$\sum_{k=0}^{N_p-1} c_{j,k} c_{q,k} = 0 \quad \forall j \neq q. \quad (36)$$

Equation (35) will reduce

$$\hat{d}_j = \sum_{k=0}^{N_p-1} c_{j,k} \frac{|h_k|^2}{\frac{1}{\varsigma_k}} d_j + \sum_{k=0}^{N_p-1} c_{j,k} \frac{h_k^*}{\frac{1}{\varsigma_k}} n_k. \quad (37)$$

### VI. Simulation results

In order to evaluate the performance of the proposed algorithm, we have compared it with the Zhang et al. algorithm, for that we have considered on theoretical channel as the Proakis’s ‘B’ channel and practical frequency selective fading channel, called Broadband Radio Access Network (BRAN C), normalized for (MC–CDMA) systems. The channels output was corrupted by an Additive Gaussian Noise for different sample sizes and for 50 Monte Carlo runs.

To measure the strength of noise, we define the signal-to-noise ratio (SNR) as:

$$SNR = 10 \log \frac{\sigma^2_{\text{SN}}(k)}{\sigma^2_{\text{N}}(k)}. \quad (38)$$
To measure the accuracy of parameter estimation, we define the mean square error (MSE) for each run as:

\[
MSE = \frac{1}{q} \sum_{i=1}^{q} \left( \frac{h(i) - \hat{h}(i)}{h(i)} \right)^2,
\]

(39)

where \(\hat{h}(i), i = 1, \ldots, q\) are the estimated parameters in each run, and \(h(i), i = 1, \ldots, q\) are the real parameters in the model.

A. Proakis ‘B’ channel

We consider the Proakis ‘B’ channel (Non-Minimum Phase) described by the following equation:

\[
y(k) = 0.407x(k) + 0.815x(k - 1) + 0.407x(k - 2),
\]

(40)
in noise free case.

\[
r(k) = y(k) + n(k),
\]

(41)
in presence of Gaussian noise.

In the Table I we represent the estimated impulse response parameters using proposed algorithm compared with the Zhang et al. algorithm.

| Table I |
|------------------|------------------|------------------|
| SNR     | Alg-ZSS         | Alg-Zhang        |
| 0 dB    | 0.3061±0.5692   | 0.0016±0.1875   |
| 8 dB    | 0.1822±0.7967   | 0.0148±0.6722   |
| 16 dB   | 0.2998±0.4851   | 0.1772±0.2225   |
| MSE     | 0.1984±0.3687   | 0.1760           |

From the simulation results, presented in Table I we observe that, For all SNR and data length \(N = 4096\), the values of MSE of the proposed algorithm are smaller than those obtained by the Zhang et al algorithm, this implies the true parameters are near the estimates parameters if we used the proposed method (Alg-ZSS). In very noisy environment (\(SNR = 0dB\)) we observe that the noise Gaussian have not influence to the developed algorithm, but, had an influence on Zhang et al algorithm. This is due to non linear of the cumulants in Zhang algorithm, or the fact that Gaussian noise higher order cumulants are not identically zero, but they have values close to zero.

The following fig. 6 give a good idea about the precision of the proposed algorithm.

To conclude, the proposed method is able to estimate the parameters impulse response of the non minimum phase channel, such as the Proakis ‘B’ channel, in noisy environments.

B. BRAN C Channel

In this paragraph, we consider the problem of Broadband Radio Access Network channel identification. In Table II we have represented the values corresponding to the BRAN C radio channel impulse response. Equation (42) describes the impulse response of BRAN C radio channel.

\[
h_c(k) = \sum_{i=0}^{N_f} A_i \delta(k - \tau_i).
\]

(42)

Although, the BRAN C channels is constituted by \(N_f = 18\) parameters and seeing that the latest parameters are very small, for that we have taking the following procedure:

- We decompose the BRAN C channel impulse response into four sub-channel as follow:

\[
h(k) = \sum_{j=1}^{3} h_j(k)
\]

(43)

- We estimate the parameters of each sub-channel independently.

- We add all sub channel parameters, to construct the full BRAN C channels impulse response.

In fig. 7 we represent the estimation of the impulse response of BRAN C channel using the proposed algorithm compared with the Zhang et al algorithm in the case of \(SNR = 16dB\) and data length \(N = 5400\).

From the fig. 7 we observe that the estimated all target of
Fig. 7. Estimated of the BRAN C channel impulse response, for an $SNR = 16dB$ and a data length $N = 5400$.

BRAN C radio channel impulse response will be closed to the true ones using the proposed method (Algo-ZSS), but if we use the Zhang et al algorithm (Algo-Zhang) we remark more difference between the estimated and real parameters.

In the following figures (fig. 8 and fig. 9) we represent respectively the estimated magnitude and phase response of the BRAN C channel using the proposed and Zhang et al algorithm, when the $SNR = 16dB$ and the data length $N = 5400$.

From the fig. 8 and fig. 9 we observe that the proposed method (Algo-ZSS) give us a very good estimation of magnitude and phase of BRAN C channel impulse response. But using (Algo-Zhang) we remark a more difference between the estimated magnitude and the measured ones. Then we observe that the estimate of the phase of BRAN C channel impulse response degrade if we used the (Algo-Zhang) algorithm.

Finally, we obtain a very good estimation of magnitude and phase response of the BRAN C channel, principally if we used the proposed algorithm (Algo-ZSS).

![Fig. 8. Estimated of the BRAN C channel impulse response using all target, for an $SNR = 16dB$ and a data length $N = 5400$](image_url)

![Fig. 9. Estimated of the BRAN C channel impulse response using all target, for an $SNR = 16dB$ and a data length $N = 5400$](image_url)

VII. MMSE EQUALIZER TECHNIQUE TO CORRECT THE CHANNEL DISTORTION

In this section we consider the $BER$, for on equalizer Minimum Mean Square Error (MMSE), to evaluate the performance of the (MC-CDMA) systems. The results are evaluated for different values of $SNR$.

We represent in the fig. 10, the simulation results of BER estimation, for different $SNR$, using the proposed algorithm (Algo-ZSS) compared with the results obtained the (Algo-Zhang) algorithm of the BRAN C channel impulse response.

From the fig. 10, we observe that the blind MMSE equalization give us approximately the same results obtained by the measured BRAN C values using (Algo-ZSS), than those obtained by (Algo-Zhang) algorithm, we have a more difference between the estimated and the measured ones. Thus, if the $SNR$ is superior to $18dB$, we observe that 1 bit error occurred when we receive $10^2$ bit with the (Algo-Zhang), but using (Algo-ZSS) we obtain only one bit error for $10^3$ bit received.

![Fig. 10. $BER$ of the estimated and measured BRAN C channel using the MMSE equalizer.](image_url)

VIII. CONCLUSION

In this contribution, we have proposed an algorithm based on higher order cumulants, in order to test its efficiency, we
have compared with (Algo-Zhang) algorithm. The simulation results show the precision of the proposed algorithm (Algo-ZSS) than those obtained using (Algo-Zhang), mainly if the input data are sufficient. The magnitude and phase of the impulse response of BRAN C channel is estimated with very important results in noisy environment principally if we use the proposed method (Algo-ZSS). In part of (MC-CDMA) systems application, it is demonstrated that the results obtained by MMSE technique equalization of the downlink (MC-CDMA) systems, using the (Algo-ZSS) is more accurate compared with the results obtained with the (Algo-Zhang) algorithm.

REFERENCES


